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THE

PHYSICAL REVIEW.

INSTABILITY OF ELECTRIFIED LIQUID SURFACES.

By John Zeleny.

IN a recent paper[1] a brief description was given of the appearance of a liquid surface undergoing disintegration owing to instability arising from an electric charge.

The observations recorded were made in connection with some experiments on the electric discharge from liquid surfaces and the work was confined to eye observations, through a microscope, of the surface in question when this was illuminated by the light of a spark from a Leyden jar. Some of the phenomena appeared to be of sufficient interest to warrant the making of a more accurate record of them by the aid of photography, and a few results obtained by this method are described in this paper.

The apparatus used for getting the electrified surface was similar to that used previously and for details reference is made to the paper mentioned. Briefly, the apparatus consisted of a vertical glass tube, 0.92 mm. in diameter, joined from its upper enlarged end by rubber tubing to a reservoir of the liquid. A drop of liquid at the lower end of the small glass tube was the part under observation. The liquid was charged to several thousand volts from a static machine, and a grounded plate was placed about 2 cm. below the end of the glass tube. Ethyl alcohol was used for nearly all of the experiments inasmuch as water is not a convenient liquid for showing some of the phenomena, because, owing to its high surface tension, the potential at which instability of its surface is first obtained in air at atmospheric pressure is nearly the same as that at which an electric discharge begins.

2. The source of light mostly employed in taking instantaneous pictures of the liquid was a condenser discharge between magnesium electrodes in air. For some of the exposures the spark was passed through mercury

[1] Proc. Camb. Philos. Soc., 18, p. 71, 1915.

vapor at atmospheric pressure in an arrangement similar to that described by C. T. R. Wilson.[1]

An induction coil was used for charging the Leyden jars, and this was provided with a rotary, mercury-jet interrupter which could be operated successfully up to 800 interruptions per second for taking moving pictures of the phenomena studied. In taking such pictures an image of the drop was focused on a vertical slot in the center of a long board placed about 2 meters from the object. A photographic film was placed in a slide on the far side of this board, and this slide, propelled by strong rubber bands, was shot past the opening in the board at a speed that carried it a distance equal to the width of the opening in the time between two interruptions. For these pictures transmitted light and the magnesium electrodes were used exclusively.

3. Some of the photographs which were taken are reproduced in Plate I. The magnification is not the same for all of the pictures but it can be estimated in each case from the diameter of the glass tube which was 0.92 mm. throughout. The liquid used was alcohol except for the case represented by Fig. 7. No luminosity accompanied any of the phenomena shown so that the so-called point discharge was not present and all transfer of electricity from the charged surface was effected by means of droplets of liquid and none by gaseous ions.

Two series of pictures taken in the manner described in § 2 are shown in Figs. 1 and 2, the time interval between the separate exposures being approximately one eight-hundredth of a second. The pictures are to be followed from right to left as the downward motion of the flying drops indicates. The doubling of some of the pictures is due to irregularity in the action of the illuminating spark. In the experiment represented by Fig. 1, the alcohol was charged to 5,000 volts and the liquid in the reservoir was 3 cm. above the end of the glass tube, while for Fig. 2 the corresponding numbers were 6,000 volts and 4 cms.

4. A few general statements may help to make clear what is going on in these pictures. Suppose the liquid unelectrified and the supply reservoir raised until liquid issues from the end of the tube at a certain slow rate. Drops with a diameter larger than that of the tube will form, break off and fall away. On electrifying the liquid sufficiently and lowering the reservoir to keep the outflow of liquid the same as before, the drops will be drawn out into more or less cylindrical form before they break from the tube. After severance these cylindrical pieces will coalesce into drops whose diameter may be considerably smaller than that of the glass tube from which they came.

[1] Proc. Roy. Soc. London, 87, p. 279, 1912.

When however the electrification is increased to a point where the electric force at the surface of the liquid attains a certain limiting value, which depends upon the surface tension of the liquid and the radius of curvature of its surface, then, irrespective of whether or not the liquid pressure is sufficient to force liquid out of the tube, the surface becomes unstable and any slight accidental displacement of the surface results in a rapid increase of that displacement. This condition is first reached at the lower end of the drop where the electric density is greatest. What happens is that the liquid at this place is pulled out into a fine thread, which eventually breaks up into minute drops.

5. Returning now to Figs. 1 and 2, it will be seen that very fine points of liquid appear on some of the drops at the end of the tube and also on some of the detached masses. These are the places where there is surface instability, although the fine threads of liquid spoken of can hardly be made out and the myriads of small droplets forming from them are quite invisible.

The fact that instability is confined to but one very small area of the surface is to be explained by the redistribution of electric charge caused by the liquid drawn out from the first place that breaks down. It is possible, however, by increasing the voltage of the surface to have a number of these places of instability existing at the same time on a surface of the dimensions used in these experiments.

As already explained the emission of the large drops seen in the figures is not an accompaniment of surface instability but is conditioned by the excessive pressure of the liquid in the tube. This pressure may be removed by lowering the supply reservoir and then the surface of the drop on the tube appears quite stationary with one or more of the fine points of liquid coming quite abruptly out of the surface where instability obtains. It is noted that the elongated detached masses of liquid retain the instability points for a short time only after they are separated from the tube above. The numerous droplets formed at the points soon carry away enough electric charge to reduce the surface to stable conditions, after which each mass of liquid quickly collapses into a spherical drop. The rapidity with which this collapse takes place is well illustrated by the first two pictures on the left in Fig. 1. In the second picture the elongated cylinder still carries the pointed end while in the first picture, only one eight-hundredth of a second later, the whole has collapsed into a nearly spherical form.

The shielding effect of the drops explains a common behavior illustrated by the pictures in Fig. 1, where it is seen that the drops fly alternately to one side and the other side of the vertical.

The electric force acting on some of the fine threads of liquid often undergoes very rapid changes in direction on account of the drops of liquid in front. Thus in the lower part of the sixth picture in Fig. 2 the fine thread appears as a large double loop, owing to the shielding effect of a drop just off the picture. Smaller but more complex loops are seen in some of the other pictures as for example in the third picture of Fig. 1. Again, the two successive pictures of Fig. 8 show a complete reversal of field. Fig. 3 shows a case where the liquid was torn into shreds by forces varying rapidly in direction. A number of the pieces are barely visible, being out of focus. The appearance at the end of the cylinder in Fig. 4 is that of clouds of finely divided material.

6. Under certain conditions of potential and hydrostatic pressure it is possible to get the alcohol drop at the end of the tube to assume the form of a cone with a fine thread of liquid coming from its apex. This condition is quite steady and is especially suited for a closer study of the liquid thread, which characterizes the state of instability, as well as of the droplets into which the thread disintegrates. The general appearance of this stage is shown in Fig. 5 which represents a picture taken after an exposure of 30 seconds with light concentrated upon the object from an arc lamp placed at the side. The potential of the drop was 5,000 volts.

The thread or stem of liquid coming from the apex of the cone actually had a much smaller diameter than the picture shows since it was not perfectly stationary during the exposure. Measurements of the thread with a microscope showed its diameter to be approximately 0.004 mm. Combining this value with a measurement of the rate of emission of the liquid it was found that the thread was pulled out at the rate of about 8 meters per second. The liquid thread remains intact for but a short distance in this case, breaking up into drops rather suddenly at the place where the enlargement shows in the picture. This enlargement is not apparent however in eye observations with a microscope, the drops flaring out gradually from the solid stem.

The spreading of the drops formed from the central thread of liquid into a more or less conical volume is most probably due to the combined action of the divergence in the electric field and of the mutual repulsion of the drops. No evidence has been obtained of any still finer threads coming from the end of the thread visible in the picture.

The line of demarcation seen in the brushlike cloud of drops shows these to be of two different sets. The outside portion is presumably made up of the set of comparatively small drops which form between the main drops whenever any liquid jet breaks up into drops. None of

FIG. 1.

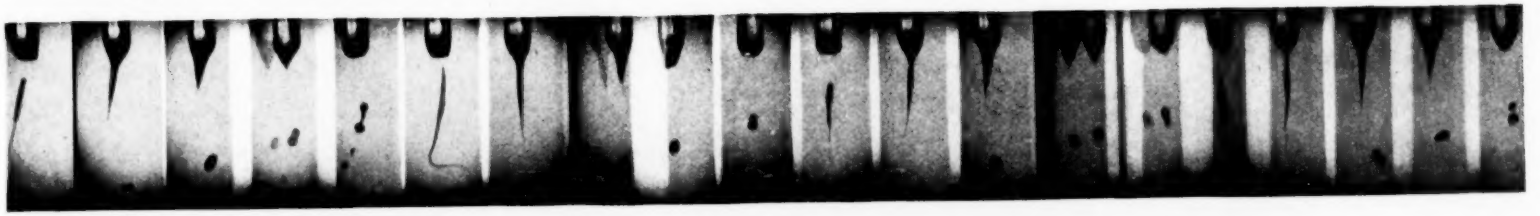

FIG. 2.

FIG. 3.

FIG. 4.

FIG. 5.

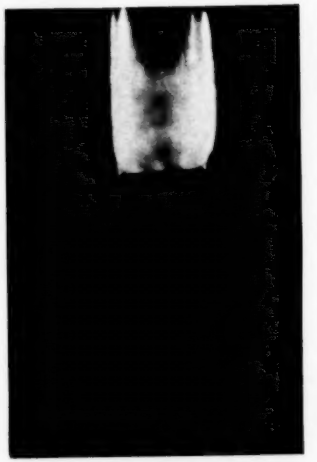

FIG. 6.

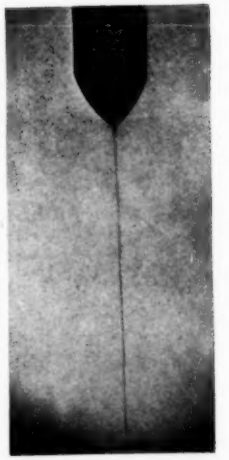

FIG. 7.

FIG. 8.

JOHN ZELENY.

the individual drops could be seen in flight with a microscope in instantaneous light, but high powers could not be used owing to their short working distances. Rapid evaporation doubtless made futile the attempts which were made to observe them after catching them on some solid surface. The measurements given above show that even if a length of thread equal to twice its diameter went into the making of each drop, the number of these drops formed per second would be a million.

It may be remarked that the brush spray appears colored both by transmitted light and by reflected light, the colors persisting in each case when the eye is within about 45° of the direction of the light. The colors depend not only on this angle of sight but differ also for the different portion of the brush, indicating thus a difference between the sizes of the drops at these parts.

It should be added that when the condition represented by Figs. 1 and 2 is viewed in continuous light it too has the appearance of a more or less compact brush, like that in Fig. 5, although the outline is not so sharp and the appearance is somewhat granular.

7. On reducing the hydrostatic pressure below the value which was requisite for maintaining the single central spray shown in Fig. 5, the cone of liquid flattens into a drop and the thread of liquid now issues from the side of this drop. Under these conditions of pressure it is possible by increasing the voltage to obtain two points of instability on the surface where two jets of liquid with their sprays of drops come from the surface, as shown in Fig. 6, which is again a time exposure by side illumination. By increasing the voltage still further eight or more such stationary jets may be obtained at the same time, the jets being all arranged on the outer edge of the drop.

8. Glycerine was used in some of the experiments because its viscosity is so much greater than that of alcohol. When the conditions were arranged for a single thread coming from a steady surface it was found that the thread was pulled out in this case a distance of 15 mm. before it underwent disintegration into drops. The set of large drops flared out in their flight into a fan similar to that obtained with alcohol (Fig. 5) but the small drops all shot out from one place in directions at right angles to the axis of the thread and the sharp outer boundary of their paths formed a paraboloid of revolution about this axis. The phenomenon was not sufficiently stationary to permit a successful time exposure to be taken by reflected light, but Fig. 7 shows a picture taken with a 2.5 seconds' exposure by transmitted light of a part of the thread coming from the drop of glycerine. The potential used was 7,000 volts. The diameter of this thread was approximately 0.007 mm. and the speed with

which it was pulled from the drop was about 3 meters per second. The diameter of the drops was found by catching them in various ways and measuring them under a microscope. The large drops in the central brush differed considerably in size but had an average diameter of about 0.01 mm., and the diameter of the small ones in the outer flare was approximately one quarter of this value.

The long known experiment of threads being pulled from highly electrified molten sealing wax is doubtless an example of the action described in this paper.

I am greatly indebted to the skill of my assistant, Mr. W. B. Lang, for the success of the pictures.

SLOANE LABORATORY,
YALE UNIVERSITY.

THE MAGNETIZATION OF IRON, NICKEL, AND COBALT BY ROTATION AND THE NATURE OF THE MAGNETIC MOLECULE.[1]

By S. J. Barnett.

§ 1. In December, 1914, I described to the American Physical Society an extended series of experiments completed in that year on the magnetization of large steel rods by mere rotation.[2]

Before these experiments were made only one method of magnetizing a body was known, viz., placing it in a magnetic field. These experiments not only revealed another and entirely new method, but they also confirmed completely the fundamental assumptions on which the results had been predicted: They proved, in a direct and conclusive way, on the basis of classical dynamics alone, without the slightest dependence upon the ill understood theory of radiation, (1) that Ampèreian currents, or molecular currents of electricity in orbital revolution, exist in iron; (2) that all or most of the electricity in orbital revolution is negative; and (3) that it has mass, or inertia, so that each orbit behaves like a minute gyrostat and tends to set itself with the direction of revolution coincident with the direction of rotation impressed on the body. It is in this way that magnetization of the body results. Furthermore, if we admit the classical theory of radiation, these experiments, together with the existence of residual or permanent magnetization, prove (4) that the arrangement of the electricity in the Ampèreian orbits is Saturnian rather than planetary.

§ 2. The theory of these experiments is given in the earlier paper already referred to. If it is assumed that only one kind of electricity is in orbital revolution, and if the mass of a particle is denoted by m and its charge by e, it is shown that the rotation of a body with angular velocity n revolutions per second is equivalent to putting it in a magnetic field of intensity H, such that

$$\frac{H}{n} = 4\pi \frac{m}{e} \qquad (1)$$

[1] A paper read before the American Physical Society, December, 1916. A brief account of this work is published in the Proceedings of the National Academy of Sciences, March, 1917, p. 178.

[2] Barnett, S. J., Phys. Rev., (2), 6, 239, 1915.

with extreme precision for all angular velocities experimentally attainable. If electrons alone are assumed to be in orbital revolution, the second member of this equation becomes -7.1×10^{-7} E.M.U. for electrons in slow motion according to experiments which are well known; and H/n should be equal to this quantity and identical for all substances. If positive electricity also participates the magnitude of H/n should be smaller. The mean value of H/n obtained in my 1914 experiments was -3.6×10^{-7} E.M.U.; and H/n was found to be independent of the speed within the limits of the experimental error.

§ 3. Not very long after my first conclusive experiments were presented to the American Physical Society, Einstein and de Haas, in February and April, 1915,[1] described to the German Physical Society successful experiments on the effect converse to mine, viz., the rotation of an iron rod by magnetization, which had been predicted and looked for by O. W. Richardson in 1907;[2] and de Haas has recently continued this work in a somewhat different manner.[3] Both investigations are indirect but excellent confirmations of my own earlier work. This work has also been confirmed by further experiments of my own of increased precision described before the American Physical Society in April, 1915.[4]

§ 4. The fundamental character of the problem, whose importance with reference to molecular constitution is rendered greater by the extreme difficulties encountered by the electromagnetic theory of radiation in attempting to account for even the simplest cases of the Zeeman effect and other allied magneto-optical effects, has led me to extend the investigation, within the last year, to other specimens of iron and to cobalt and nickel. In all the earlier work the method of electromagnetic induction was used, a fluxmeter being the principal measuring instrument. The new work described in this paper has been done by the method of the magnetometer. It is more difficult to eliminate extraneous disturbances with the new method than with the old, but it is less difficult to attain adequate sensibility without the use of large rods, the cost of which, in the case of cobalt and nickel, would be great; moreover, on account of the complete novelty of the effect under investigation, it was considered desirable to use a method as nearly independent of the earlier one as possible.

§ 5. The magnetometer was an astatic instrument, and the rod under

[1] Einstein, A., and de Haas, W. J., Verh. d. D. Phys. Ges., 17, 152, 203, 420, 1915.

[2] Richardson, O. W., PHYS. REV., (1), 26, 248, 1908.

[3] de Haas, W. J., Science Abstracts A, 17, 351, 1916. The original paper has not yet reached me.

[4] Barnett, S. J., *loc. cit.*

experiment, or rotor, was mounted with its axis horizontal and normal to the magnetic meridian in the equatorial position of Gauss, which offered important advantages for this work.[1] Calibrations were made by means of solenoids wound permanently on the rotors and subsidiary solenoids wound on wooden cores. Rotation observations were made at equal intervals of time in sets of four as follows: The rotor was first driven (by means of an alternating current motor) at a determined speed in one direction, and the magnetometer scale read; then the motor was reversed and the scale again read for the same speed; then the readings were repeated in inverse order. From the double deflection obtained by subtracting the mean of the second and third readings from the mean of the first and fourth, together with the angular velocity of the rotor, and the calibration experiments, the quantity H/n could be determined. The details of the experimental work and the means used to eliminate extraneous disturbances are described below.

§ 6. Diagrams of important parts of the apparatus, drawn approximately to scale, are given in Fig. 1, and reproductions of actual photo-

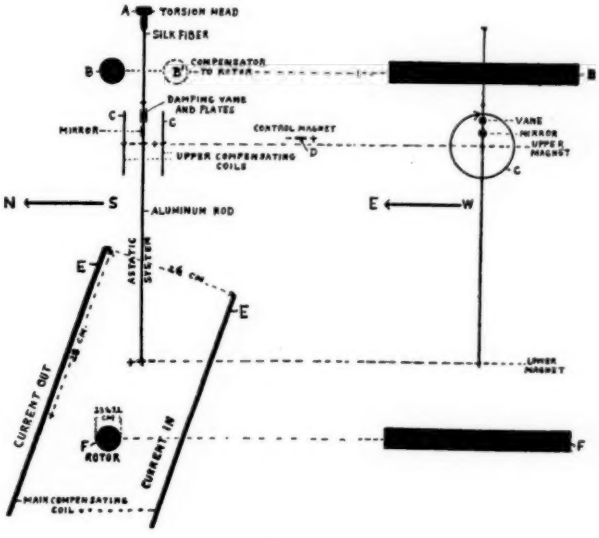

Fig. 1.

graphs are given in Figs. 2, 3 and 4. All the figures have been lettered to correspond. In the earlier part of the work each magnet of the astatic system was carefully made of three small pieces of tungsten steel. In the rest of the work each magnet was made of eight steel cylinders of very nearly the same length. All were cut from the same wire and

[1] Adoption of the polar position would have made it extremely difficult to make satisfactory calibrations and to eliminate sources of serious error.

hardened together. The two groups were magnetized in the same field after being mounted properly on the light aluminum rod shown in the figure. This rod carried also a small plane mirror and a thin aluminum damping vane, and was suspended from a torsion head A by a single silk fiber. The complete suspension was mounted in a groove milled in the bronze casting M, with enlargements cut for the mirror, damping vane, and two adjustable parallel damping plates. The long groove was covered with a strip of brass; and an opening for the mirror and two openings for observation of the damping arrangement were covered with glass. The enclosure was sealed with universal wax to prevent air currents. The casting M holding the magnetometer system was screwed to a heavy ribbed H-form bronze casting L. At its four corners the casting L was bolted to bronze cones sunk into the tops of the four concrete pillars K, K, K', K', cemented to the concrete floor. To make the mounting more rigid, heavy boards extending from arm to arm were screwed onto the lower surface of the H-form casting.

§ 7. Numerous experiments were made with four different rotors of the type and dimensions indicated in Fig. 5. In constructing each rotor

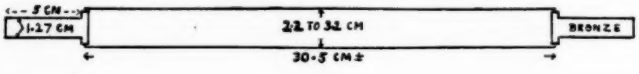

Fig. 5.

the magnetic material was first turned to the shape indicated by the central portion of the figure, except in the case of cobalt, where there was a slight difference; then the bronze bearing pieces, previously turned to the shape indicated but with centers at the ends, were soldered to the ends of the magnetic material. Then the complete structure was centered in the lathe and all the surfaces turned true to the same centers. One end of a fine insulated copper wire was then soldered to one end of the magnetic material and the wire was wound over its surface on the lathe into a solenoid with 16 turns to the inch. The wire and metal surface were then given a heavy coat of shellac and dried. Several centimeters of the free end of the solenoid were then wound with insulating tape and covered with several layers of tin foil. The free end of the copper wire was then stripped of its insulation and bent over this tin foil, and a number of additional layers wound on. Then the ends of the foil were thoroughly secured to the rotor with insulating tape. In calibrating experiments the tin foil and the bronze bearing piece near it were used as terminals. Before calibrating, the resistance of the solenoid was measured to make sure that no short-circuit existed.

Two of the rotors were of cold-rolled steel shafting: One of them

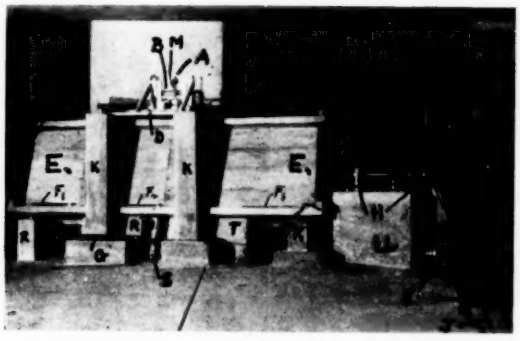

Fig. 2.

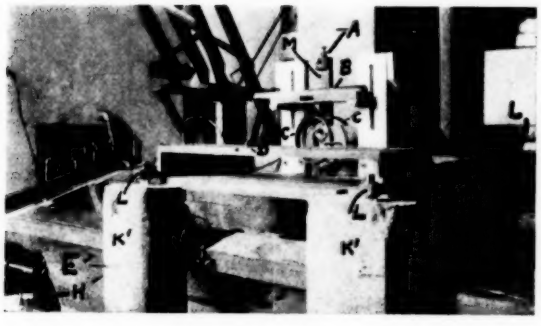

Fig. 3.

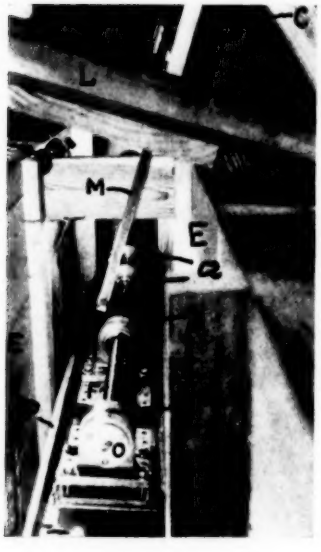

Fig. 4.

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was about 2.3 cm. in diameter and 30.6 cm. long; the other about 3.1 cm. in diameter and 30.4 cm. long. One of the rotors was nickel, about 2.2 cm. in diameter and 30.6 cm. long. Another rotor was of cobalt about 3.2 cm. in diameter and 30.4 cm. long. The main surface of the cobalt was a true cylinder like those of the other rotors, except that three shallow grooves which had been turned into it were filled with thin brass bands soldered in. Also, the cobalt casting was somewhat imperfect, being pitted with small holes. A fifth rotor, of Norway iron, was constructed like the others, except that a washer was added at each end—a fact which I discovered after finding that it failed to give satisfactory results. Only a few rough observations were made with this rotor.

Three of the rotors are shown at F_1, F_2 and F_3 in Fig. 2, and one of shown in its bearings at F in Fig. 4.

§ 8. The rotor moved in cylindrical lumen bearings, one of which, O, is visible in Fig. 4. These lumen bearings were screwed into bronze holders, themselves bolted into bronze castings NN, Fig. 4. The castings NN were bolted to a single casting of bronze, which was bolted to a heavy bronze bed plate P. The casting P was bolted to bronze cones sunk into the oblique concrete piers S and T, Fig. 2. To assist in reducing vibration, a considerable portion of the space between the casting P and the piers was filled with cement and the plate bolted down before the cement hardened. The magnetic meridian through the magnetometer magnets passed nearly, but not exactly, through the center of the rotor.

§ 9. The rotor was driven by a brass rod about 0.6 cm. in diameter and 24 cm. long from a small bronze shaft with lumen bearings mounted in bronze castings Q, Fig. 3, on the bed plate P. This countershaft was itself driven by a brass rod about 0.6 cm. in diameter and 42 cm. long from a larger bronze countershaft mounted with lumen bearings in brass and bronze castings H, Figs. 2 and 3, on the concrete pier U. The west end of this larger countershaft carried a three speed bronze pulley by means of which and a similar pulley J, Fig. 2, on the electric motor, and a round belt, it was driven at speeds near to 20, 30 and 45 revolutions per second. In the earliest part of the work another arrangement was used giving speeds over 50 revolutions per second, but this was soon discontinued.

The electric motor was a one horse-power Century alternating current single phase motor, and gave excellent satisfaction. On constant supply it gave constant speeds which were identical for both directions of rotation. It was reversed from a distance by simply pulling one of two

strings fastened to a lever which was attached to the brush holder. Speeds were determined with a very small direct current dynamo, separately excited at constant voltage by a storage battery, and a millivoltmeter connected through a high resistance and a reversing switch with the brushes. A pulley on the armature was driven by a long belt, *J*, Fig. 2, from a pulley on the main countershaft driving the rotor. The voltmeter readings for the same speed differed slightly for the two directions of rotation, and depended slightly on the temperature of the field coils. In obtaining the speed from the voltmeter readings, these effects were allowed for.

§ 10. To compensate as far as practicable for disturbances produced by variations in the earth's magnetic intensity, a rod *B*, Figs. 1, 2 and 4, called a compensator, was used. It was of the same material as the rotor, since both were cut from the same rod in the case of each substance investigated, and of nearly the same size, and was mounted in approximately the same position with respect to the upper magnetometer magnet as that occupied by the rotor with respect to the lower magnet. Usually the compensator was placed in approximately one of the symmetrical positions *B*, *B'*, Fig. 1, but the best position had to be found by trial.

§ 11. Although the earlier investigation on iron by the method of electromagnetic induction had shown that the rotation of the rotor in the earth's magnetic field gave the same or nearly the same results as were obtained when the intensity of this field was annulled by a suitable electric coil, it was considered important for the present investigation to provide means of neutralizing the earth's field throughout the region occupied by the rotor. For this purpose the large and accurately made coil, of rectangular cross-section, used in the last part of the earlier investigation was slightly modified. The frame work was shortened along the axis of the coil, and strengthened and made still more nearly true by brass bolts and internal wooden braces near the central section. The coil was reduced to 56 cm. in length, and was left about 26 cm. wide and 198 cm. broad. The coil and frame are marked with the letter *E* in Figs. 1–4.

As in the earlier experiments, the coil was mounted over the rotor and its oblique piers *S* and *T*, the centers of coil and rotor being made nearly coincident. The position of the frame was adjusted until the long edges were horizontal and perpendicular to the magnetic meridian, and the axis of the coil was parallel to the earth's intensity. Then the frame was bolted to bronze cones sunk into six concrete piers, three on each of the larger sides. The three piers on the north side are shown at *R R R* in Fig. 2.

After the coil had been clamped in position, with the heavier bronze castings in place, its magnetic field, when it was traversed by an electric current, was studied throughout a region including and extending somewhat beyond that to be occupied by the magnetic part of the rotors. This field was found to be uniform to one part in five hundred. By a method similar to that used in the earlier experiments, it was found that a current in the proper direction giving 91.4 divisions (equivalent to about 0.389 ampere) on a Weston instrument (with special shunt) compensated completely the earth's flux through a steel rod 33 cm. long and 3.2 cm. in diameter with center in the position to be occupied by the center of each rotor. The compensation was sensitive to a tenth division, or about one part in nine hundred. After the rotation experiments were completed the compensation was again tested by the same method and was found to have remained unaltered. The current was kept at the compensating value 91.4 divisions during nearly all of the observations, and was always kept within one tenth, or in a few instances two tenths, of a division of that value, except for testing purposes as indicated below.

The concrete piers and all the castings near the rotor were free from iron, and all the other bronze and brass castings were either free from iron or so nearly free that any effect on the field in the region occupied by the rotor was quite negligible.

§ 12. Since the lower magnetometer magnet hung, as seen from Fig. 1, in a region in which the earth's intensity was nearly annulled, it was necessary, in order to keep the magnetometer sensibility and zero reading approximately independent of the current in the compensating coil, to provide special coils *CC*, Figs. 1–4, to compensate approximately the horizontal component of the earth's intensity in the region occupied by the upper magnet. Each of these coils contained three turns of insulated wire. They were wound in vertical planes on brass rings 10 cm. in diameter, whose bases were moved for adjustment in brass slides by a right and left handed screw in such a way that the magnet was always approximately at the center of the system. The coils were connected in series with the main compensating coil, and both were connected to oppose the earth's intensity. The distance between the rings was adjusted in the different experiments until the earth's horizontal intensity at the upper magnet was nearly annulled.

The control magnet *D*, Figs. 1–3, was a small piece of hardened tool steel.

With the arrangement described the sensibility of the magnetometer was not altered when the compensating current was changed by one part

in ninety, from 91.0 to 92.0 divisions on the indicating instrument, and the zero was altered but little.

§ 13. The magnetometer was almost always used with approximately critical damping, when it usually required from 15 to 20 seconds to reach its elongation.

The mirror was a small plane mirror and the opening in front of it in the bronze holder was covered with a convex spectacle lens. Deflections were read to tenths of millimeters by means of a single filament tungsten nitrogen-filled lamp and a translucent scale, distant about 4 m. from the mirror except in the very earliest part of the work.

§ 14. Calibration experiments were of two kinds. At the beginning and the end of a series of observations, except in rare instances, the approximate sensibility was carefully determined by the process described below; and on a single occasion for each rotor the correction necessitated by the presence and finite length of the solenoid permanently wound upon it was determined once for all. This correction was much less than the experimental error, but was nevertheless always made.

The approximate calibration for each series of rotation experiments was made as follows: A dry cell in good condition, with open circuit E.M.F. 1.50 volt, was connected through a suitable key and a standard high resistance—7,500 to 25,000 ohms—in series with the solenoid of the rotor under experiment, and magnetometer deflections on opening and closing, or (usually) double deflections on reversing, the key were obtained. Each solenoid, as stated above, was wound by lathe with 16 turns to the inch. If D denotes the double deflection, R the box resistance in the circuit (that of battery and solenoid being negligible) in ohms, and h the magnetic intensity which the solenoid, if very long, would impress on the rotor, and if d denotes the double deflection produced on reversing the rotor in the rotation experiments, the intrinsic magnetic intensity of rotation is approximately

$$H = \frac{d}{D}h = \frac{4\pi \times 16 \times 1.50\, d}{10 \times 2.54 \times R\, D} \text{ gauss;}$$

and, if n denotes the rotor velocity in revolutions per second, the intrinsic intensity per unit velocity is approximately

$$\frac{H}{n} = \frac{4\pi \times 16 \times 1.5 \times d}{10 \times 2.54 \times R \times D \times n} \frac{\text{gauss}}{\text{rev. per second}}.$$

Since the magnetometer zero and sensibility depended on the position angle of the rotor, the mean value of D for three position angles differing successively by 120° was obtained in all the later experiments. The same value of the mean was obtained for three position angles half way between those just mentioned. In the case of the earlier observations

on the larger rotor of steel and the rotor of cobalt, the omission of this precaution introduced a possible error of 2 or 3 per cent., which, however, is much less than the experimental error in the rotation experiments. In the case of the nickel rotor the error introduced in this way was only a half of one per cent., or less; and the same thing would be true of the smaller steel rotor, which, however, was always calibrated with the three or six position angles.

The calibrations at the beginning and end of a series always agreed closely. All calibrations were made with the proper current in the compensating coils, although, as already stated, considerable variations of the compensating current did not effect the sensibility appreciably.

§ 15. The experiment to correct for the departure from uniformity of the field produced by the rotor's solenoid and for the effect of the solenoid itself were most conveniently made with the rotors and coils near the upper magnet instead of the lower magnet.

In the case of each rotor it was found that adding to each end a solenoid 10 cm. in length, wound like the rotor but on a wooden core of approximately the same diameter, made no difference in the deflection produced by a given current.

The total length of the combined solenoids was about 50.5 cm. Solenoids of approximately the same diameters as those of the rotors and wound in the same way, but on wooden cores 50.5 cm. in length were mounted symmetrically in the place occupied previously by the rotors, and the deflections produced by known currents observed. From the ratio of the currents, and the ratio of the magnetometer sensibilities with rotor present and rotor absent, which precautions were taken to obtain, the ratio of the deflection produced by the solenoid alone to that produced with the same current by the solenoid and the rotor together was obtained for each rotor. The corrections thus found were 1.2 per cent. for cobalt; 0.6 per cent. for nickel; 0.9 per cent. for the smaller steel rod; and 1.3 per cent. for the larger steel rod.

Experiments were also made as a check and as a matter of interest with the central 30.5 cm. of the solenoids wound on wooden cores in place of the full lengths. Corrections obtained in this way are almost exactly twice those with the larger solenoids, which, being the true values, were applied to the observations.

§ 16. If C denotes the per cent. correction obtained above, and D_0 the corrected calibration double deflection, the true value of H/n will be obtained by substituting for D in the final equation of § 14 the quantity

$$D_0 = D\left(1 - \frac{C}{100}\right).$$

Thus we get, to a close approximation,

$$\frac{H}{n} = \frac{4\pi \times 16 \times 1.50 \times d \times \left(1 + \frac{C}{100}\right)}{10 \times 2.54 \times R \times n \times D} \frac{\text{gauss}}{\text{rev. per second}}. \quad (2)$$

§ 17. The chief results of the observations are given in Table I. The sets (§ 5) are arranged in groups, each group containing from 2 to 14 sets, all at very nearly the same speed. The last two columns contain, for the series of observations occupying each horizontal row, the average departure of a single set from the mean value given in column 6 reckoned in two different ways. The value in column 6 is the weighted arithmetic mean calculated by assigning to the mean for each group a weight proportional to the number of sets in the group.

To obtain the departure given in the next to the last column the procedure was as follows: For each group of the series the average departure from the group mean was determined. This was multiplied by the number of sets in the group, and the sum of the products so obtained for all the groups in the series was divided by the total number of sets in the series. The departure given in the last column was obtained by taking the difference between the mean value given in column 6 and each of the group means, multiplying each difference by the number of sets in the group, adding and dividing by the total number of sets. The two columns together give a sufficiently good idea of the experimental errors.

§ 18. In addition to the observations given in the table, a few observations were made with the larger steel rod at lower speeds when conditions

TABLE I.

Intrinsic magnetic intensity of rotation in iron, nickel and cobalt.

Rotor.	Series.	Groups.	Mean Speed R.P.S.	Number of Sets.	$-\frac{H}{n} \times 10^7$ E.M.U. Mean.	Average Departure from Mean (Sets).	Average Departure from Mean (Groups).
Steel (smaller) .	1	1–2	44.8	21	5.1	0.5	0.5
Steel (larger) . .	2	3–4	47.8	21	5.2	1.2	0.6
Cobalt	3	5–7	20.2	17	4.8	2.2	2.2
	4	8–11	30.3	23	5.6	1.2	1.4
	5	12–25	45.5	79	6.0	0.9	0.8
	6	22	45.0	7	6.5	0.3	
	7	24	44.8	9	5.9	0.4	
	8	25	44.8	5	6.1	0.4	
Nickel	9	26	20.5	4	4.7	2.0	
	10	27–28	30.5	9	6.7	1.1	1.1
	11	29–32	45.3	37	6.1	0.5	0.9

were such that the extraneous disturbances largely masked the effect being looked for. Moreover, a few sets were taken with a fifth rotor, of soft Norway iron, which gave results consistent with those given in the table, but with large discrepancies in the magnitude of the deflections. This rotor was very troublesome, and was found, after the experiments were completed, to have been constructed differently from the others—accurate washers having been added at the ends because the rotor had been found too short to fit the bearings perfectly. This construction may explain a part, but will not probably explain all, of the rotor's behavior, which has not yet been adequately investigated. Furthermore, on one occasion several sets with the nickel rod gave discordant deflections several times as great as the normal deflections for the same speed. On examination it was found that the rotor had been improperly mounted with large longitudinal play, suggesting such an effect as is obtained by tapping an iron rod while in a magnetic field. On another occasion, just after the completion of a long and good series of observations on nickel, with normal compensating current, at the end of a night's work, three sets of readings were taken with the compensating current above and below normal value. All were discordant and abnormally low. These and the other observations mentioned were omitted from the table.

§ 19. Six of the observations in series 11 on nickel were made with compensating current above and below the normal value, at 91.0 and 92.0, instead of 91.4, divisions. They are all included in the table, however, because the alterations of the current produced no change. With the rods of iron and cobalt the change produced by altering the current as in the case of nickel, from 91.4 to 91.0 and 92.0 divisions, was but little if any greater than the experimental error; but the observations are not included in the table because of the difference. In the case of cobalt the observations were taken on the same occasion with those for group 25. Group 25, with current 91.4 divisions, gave the deflection 1.00 ± 0.06 cm. for 5 sets. The same number of sets for currents 91.0 and 92.0 gave deflections 1.13 ± 0.04 cm. and 0.96 ± 0.06 cm., respectively.

§ 20. The observations mentioned in the last section show that no appreciable systematic error was introduced on account of currents induced in the rotor by its motion in the field of the earth, compensated as it was to about one part in nine hundred.

The field intensity produced at the center of the rotor by the control magnet was about one one thousandth the earth's intensity. The intensity produced by the lower magnetometer magnet at the same point was

about equal, and had always a large component opposite, to that due to the control magnet. The intensity at the rotor due to the upper compensating coils was about one three thousandth the earth's intensity; and that due to the magnetization, both permanent and temporary, of the compensating rods, was also negligible.[1] Hence it would be unreasonable to suppose that any appreciable systematic error was produced by the motion of the rotor in these fields. That no great error of this sort was introduced is proved experimentally, moreover, by the agreement of the results obtained with the two rotors of iron of different diameters, inasmuch as any eddy current effect would depend upon the diameter.

§ 21. Another possible systematic error which had to be avoided was the error arising from the shift of the rotor's axis in azimuth or altitude, the shift being probably different for the two directions of rotation. If the residual field intensity normal to the axis of the rotor is Z and the maximum angular shift possible on reversal α, the maximum change of longitudinal intensity impressed on the rotor by an angular displacement is $Z\alpha$. The difference between the internal diameter of the lumen journals and the diameter of the bronze bearings was about 0.004 cm., and the distance between the far ends of the journals was about 35 cm. For the maximum possible value of α these data give $(2 \times 4)/35{,}000$. If we assume that Z is as great as 1/500 the earth's intensity, or about 0.6/500 gauss, we obtain as the maximum value of αZ the quantity $(8 \times 6)/(35{,}000 \times 5{,}000)$ gauss, or about 3×10^{-7} gauss. This intensity, which is certainly greater than any intensity of the sort which could have been produced, is only about one fourth the change of intensity which would be produced in the rotation experiments, by reversing the direction of rotation at a speed of one revolution per second. Any such

[1] Before beginning experiments with the smaller iron rotor, and before making the later experiments with the other rotors, their compensating rods were heated to whiteness and otherwise treated to demagnetize them as thoroughly as practicable. On making tests with a magnetometer, after the rotations were concluded, it was found that the maximum magnetic intensity which the temporary diametral magnetization of any of the compensators, placed in the undisturbed field of the earth, produced at a distance somewhat less than the normal distance between compensator and rotor was about one sixteenth hundredth of the earth's intensity. The actual intensity produced in the region occupied by the rotor during the rotation experiments was much less than this, since the compensator was then in a field of reduced intensity and since the plane containing the axes of rotor and compensator made a considerable angle with the earth's intensity. The maximum intensity in the region occupied by the rotor due to the permanent diametral magnetization of any compensator was found to be less than one ten thousandth of the earth's intensity; and the maximum intensity due to the permanent longitudinal magnetization was found to be about the same. The agreement between the results obtained before and after the compensators received the treatment described shows that the effects of the compensators were negligible in the early part of the work as well as in the later.

effect in these experiments was therefore negligible. So far as angular displacements of the rotor's axis in the plane parallel to the largest side of the compensating coil are concerned, this is also proved experimentally by the observations mentioned in § 19.

§ 22. Possible systematic errors due to the longitudinal motion of the rotor, carrying its magnetization with it and undergoing changes of magnetization on account of the space variation of the longitudinal components of the residual field intensity, were avoided by mounting the rotor free from appreciable longitudinal play and observing always only the effect of reversing the angular velocity. For a given speed there is no reason to expect a different longitudinal displacement, if any should occur, on reversal of the direction of rotation. Error due to the bodily motion of the magnetization with the rotor would also be eliminated in part by the process mentioned in the next section.

§ 23. As follows from the earlier investigation on iron no error due to torsion was to be expected. Nevertheless, the rotors were made reversible in their bearings, and in the cases of nickel and cobalt many sets of observations were made with the rotor turned in each direction, a process which would eliminate the torsion error if existent. No difference was found.

§ 24. Although the rotors were demagnetized until the residual longitudinal magnetization was in no case greater than about one tenth that of the principal rod of iron used in the earlier investigation, it always happened that when a rotor was rotated very slowly by hand the image of the lamp filament moved up and down on the scale—over many centimeters in the case of the cobalt and Norway iron rotors, and over smaller ranges in the case of the others. This is one of the reasons for the necessity of always making observations while the rotor was in motion. The procedure adopted of obtaining the difference of scale readings for both directions of rotation at the same speed avoided difficulty from this source. This procedure also avoided error due to the change of magnetization by centrifugal expansion of the rotor, discovered in iron in the course of the earlier investigation. This effect was doubtless much smaller in these experiments than in the earlier ones, as the residual magnetization was much less, but the method of observation did not permit its examination.

§ 25. All bearing parts were carefully turned and adjusted, and were oiled almost invariably before each set of four readings. These precautions, with the heavy mountings and special method of driving already described, eliminated almost completely, if not completely, mechanical disturbances due to the rotation. Other mechanical disturbances

and magnetic disturbances were reduced as much as possible by beginning work in nearly all cases after one o'clock, and quitting before four o'clock, or sometimes a little later, in the night. Except during a few sets on one night, when a high wind was blowing, mechanical disturbances were never troublesome; but magnetic disturbances, in spite of the precautions taken to secure an astatic system and to adjust the magnetic compensator, were always present; and they account for the chief part of the accidental experimental error. The temperature during the night work was usually very nearly constant, and the compensating current varied very slowly, often requiring no adjustment for many sets. The speed of the driving motor also remained very nearly constant.

§ 26. In order to avoid all extraneous disturbances as far as possible the method of observation already described (§ 5) was adopted and was carried out on a regular time schedule. All being in readiness, the motor was started at a certain time T. After a fixed interval of t seconds (usually either 15^s or 20^s) the magnetometer scale and speed voltmeter were read, and the motor then stopped and the motor and voltmeter switches thrown for reversal. At the time $T + 1^m$ the motor was started in the opposite direction, and the readings taken t seconds later as before. Then observations for the two directions of rotation were made in inverse order, the motor being started at the times $T + 2^m$ and $T + 3^m$, and the readings being taken in each case t seconds later. The magnetometer double deflection obtained by subtracting the mean of the second and third scale readings from the mean of the first and fourth was independent of any slow drift and corresponded to the mean of the four speeds, always close together. In a few sets the constant interval between successive observations differed from 1^m; in a few the interval between the second and third differed from the other intervals, which was legitimate; and in some cases sudden magnetic disturbances made it necessary to observe the scale at a time differing from the schedule time; but the usual procedure was that given above, and departures from it were unimportant.

§ 27. With nickel and cobalt observations were made at three speeds. As shown in Table I., H/n was found to be independent of the speed within the limits of the experimental error, a result already obtained in the earlier experiments with iron. Since the chief disturbances were magnetic, the observations at lower speeds were less precise than those at the highest speeds. The results at the highest speeds are given in series 1, 2, 5 and 11. Series 6, 7 and 8 are a part of series 5, viz., the last three groups of results obtained with cobalt in a neutral field, one group of 5 sets (group 23), obtained shortly before group 24 while a

strong wind was blowing and the magnetometer was imperfectly damped, being excepted.

§ 28. Every set of observations[1] gave the sign of H/n negative like that of $4\pi m/e$ for an electron. The mean magnitude of H/n is in all cases somewhat less than the accepted magnitude of $4\pi m/e$, viz., 7.1×10^{-7} E.M.U., obtained from other experiments on electrons in slow motion, ranging from 5.1 to 6.5×10^{-7} E.M.U. for the most reliable observations in Table I., viz. those at the highest speeds. The differences are in the same direction as in the earlier experiments on iron, which gave 3.6 and 3.1 in place of 7.1; but the experimental errors, on account of the great difficulties involved, are such that importance cannot in my opinion be attached to the discrepancies. The investigation must rather be taken as confirming equation (1) both qualitatively and quantitatively on the assumption that only electrons are in orbital revolution in the molecules of all the substances investigated. It shows moreover that the effect is independent of the size of the body in rotation, which is an implicit requirement of equation (1).

§ 29. This investigation has been made with the aid of a grant from the university for which I am indebted to the interest of the dean of the graduate school, Professor Wm. McPherson. I am indebted to Mr. Arthur Freund, mechanician in this laboratory, for most of the finer mechanical work necessary; and I am indebted to Mrs. Barnett for a great deal of help in making the experiments.

The Physical Laboratory,
Ohio State University,
March 13, 1917.

[1] Except one, at a low speed, among the early observations mentioned in § 18, in which the discrepancies were great and the effect was reached by extraneous disturbances.

THE THERMOPHONE AS A PRECISION SOURCE OF SOUND.

BY H. D. ARNOLD AND I. B. CRANDALL.

THE acoustic effect accompanying the passage of an alternating current through a thin conductor has been known for some time, but as far as we are aware, no use has been made of the principle involved for the production of a precision source of sound energy, or standard phone. In 1898 F. Braun[1] discovered that acoustic effects could be produced by passing alternating currents through a bolometer in which the usual direct current was also maintained. An article by Weinberg[2] describes the old experiments of Braun, and also some experiments of Weinberg, in which acoustic phenomena were observed with other electrically heated conductors, rheostats, etc., through which large alternating currents were passed. A more recent application of the same principle is described by de Lange[3] in his article on the thermophone.

The writers have found that the thermophone together with a suitable supply of alternating current can be used very conveniently as a precision source of sound energy. On account of the fact that the published material on this electrical-acoustic effect is largely of a qualitative character it has been necessary to work out a quantitative theory; and it is the purpose of this paper to give the theory and show how the instrument can be adapted to acoustic measurements.

When alternating current is passed through a thin conductor, periodic heating takes place in the conductor following the variations in current strength. This periodic heating sets up temperature waves which are propagated into the surrounding medium; the amplitude of the temperature waves falling off very rapidly as the distance from the conductor increases. On account of the rapid attenuation of these temperature waves, their net effect is to produce a periodic rise in temperature in a limited portion of the medium near the conductor, and the thermal expansion and contraction of this layer of the medium determines the amplitude of the resulting sound waves. To secure appreciable amplitudes with currents of ordinary magnitude it is essential that the con-

[1] Ann. der Physik. 65, 1898, p. 358.

[2] Elektrot. Zeit. 28, 1907, p. 944. See also A. Koepsel, Elektrot. Zeit. 28, 1907, p. 1095.

[3] Proc. Royal Soc. 91A, 1915, p. 239.

ductor be very thin; its heat capacity must be small, and it must be able to conduct at once to its surface the heat produced in its interior, in order to follow the temperature changes produced by a rapidly varying current.

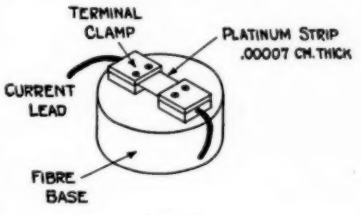


Fig. 1.

Simple Thermophone.

A simple form of instrument which we have used is shown in Fig. 1. There are two ways in which the strip may be supplied with electrical energy in order to produce sound waves, (*a*) with pure alternating current and (*b*) with alternating and direct current superimposed. If an alternating current $I \sin pt$ is supplied, the heating effect is proportional to

$$RI^2 \sin^2 pt = \frac{RI^2}{2}(1 - \cos 2pt), \tag{1}$$

so that the acoustic frequency is double the frequency of the applied alternating current. If it is desired to make the acoustic wave follow the alternating current wave, without introducing the double frequency effect, resort must be had to a superimposed direct current whose value is several times as large as the maximum value of the alternating current. If a direct current I_0 and an alternating current $I' \sin pt$ are used to heat the strip, the heating effect is proportional to

$$\begin{aligned} R(I_0 + I' \sin pt)^2 &= RI_0^2 + 2RI_0I' \sin pt + RI'^2 \sin^2 pt \\ &= R\left(I_0^2 + \frac{I'^2}{2}\right) + 2RI_0I' \sin pt - \frac{RI'^2}{2}\cos 2pt \end{aligned} \tag{2}$$

from which it is evident that the double frequency term can be made negligible by suitable choice of I_0 and I'.

When pure alternating current is used, the mean temperature of the strip is determined by the term $\frac{1}{2}RI^2$; when direct current is used with the alternating current, the mean temperature is determined by the term RI_0^2. The mean temperature of the conductor is one of the factors which sets a limit on the maximum amount of electrical energy used and hence on the maximum amount of acoustic energy that can be obtained. If only a small quantity of alternating current energy of suitable frequency is available, it is clear, from a comparison of equations (2) and (1) that more acoustic effect will be realized if direct current energy is added up to the limit that the strip will bear; for example, if I_0^2 is as large as I'^2, the product term in (2) is four times as large as the second term in (1).

Suppose now that an indefinite quantity of alternating current energy of any frequency is at hand; we desire to find the most effective way to actuate the element. Equating the terms in (1) and (2) which are proportional to the (limiting) mean temperature in each case

$$\left[\frac{RI^2}{2}\right]_{\max} = \left[R\left(I_0^2 + \frac{I'^2}{2}\right)\right]_{\max} \tag{3}$$

we can compute the maximum amplitude $2RI_0I'$ of the product term in (2) and compare this with the amplitude $RI^2/2$ of the periodic term in (1). The maximum value of the product $2RI_0I'$, consistent with condition (3) is $RI^2/\sqrt{2}$ and implies the relations

$$I = \sqrt{2}\, I' = 2I_0.$$

The amplitude $RI^2/\sqrt{2}$ in the second case is only slightly larger than the amplitude $RI^2/2$ which we should have according to (1); and in the second case there is the double frequency term of amplitude $RI^2/4$ which in most cases would be inconveniently large.

The conclusion from these calculations is that for sounding a pure tone of a given frequency it is better to actuate the strip wholly by alternating current of half that frequency. However, if it is desired to make the sound waves reproduce the electrical waves in both frequency and form, it is necessary to use in addition a direct current whose relative value is large. In this case the thermophone element is worked somewhat below maximum efficiency for the sake of minimizing the double-frequency effect.

Using the first method of excitation, it is necessary, if a pure tone is desired, that the alternating current used be a pure sine wave, absolutely free from harmonics. In order to show the acoustic effect of harmonics in the alternating current supply, consider an exciting current of the form

$$\sum_{k=1}^{n} a_k \sin kpt.$$

The heating effect produced is proportional to

$$\left(\sum_{k=1}^{n} a_k \sin kpt\right)^2 = \sum_{k=1}^{n} a_k^2 \sin^2 kpt + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} \sum_{k=2}^{n} a_j a_k \sin jpt \sin kpt$$

$$= \sum_{k=1}^{n} \frac{a_k^2}{2} - \sum_{k=1}^{n} \frac{a_k^2}{2} \cos 2kpt + \sum_{\substack{j=1 \\ j\neq k}}^{n-1} \sum_{k=2}^{n} \frac{a_j a_k}{2} \cos (j-k)pt$$

$$- \sum_{\substack{j=1 \\ j\neq k}}^{n-1} \sum_{k=2}^{n} \frac{a_j a_k}{2} \cos (j+k)pt$$

which shows that two series of combination-tones result in addition to the series of tones whose frequencies are double those of the applied fundamental and harmonics. One particular case is of practical importance: the case in which the alternating current wave consists of a fundamental and an appreciable second harmonic. In this case, besides the tones of double and quadruple frequency there are combination tones of *single* and triple frequency, a paradoxical result that is very easily verified by experiment. The importance of a pure alternating current supply is clear from the considerations given.

THE PERIODIC TEMPERATURE CHANGE IN A THIN FLAT CONDUCTOR SUPPLIED WITH ALTERNATING CURRENT.

Consider first the case of a strip supplied with both direct and alternating current. Equating the rate of production of heat by the electric current to the rate of transfer of heat to the surrounding medium, plus the rate of storage of heat in the strip, the fundamental equation may be written:

$$0.24(I_0 + I' \sin pt)^2 R = 2a\beta T + a\gamma \frac{dT}{dt}, \qquad (4)$$

in which the unit is the calorie per second, and the constants are chosen as follows:

I_0 = direct current in amperes.

I' = maximum value of A.C. in amperes.

p = $2\pi f$; f = frequency.

R = instantaneous resistance of the strip.

T = temperature of strip above surroundings.

a = area of one side of strip.

β = the rate of loss of heat per unit area of the strip (due to conduction *and* radiation) per unit rise in temperature of the strip above that of its surroundings; it is equal to the product of the temperature gradient per degree rise, into the conductivity of the medium. It can be determined experimentally, and is a constant if only conduction is considered; if it is desired to take account of radiation a modified value of β for any value of T may be obtained which is sufficiently accurate for the purposes of calculation. The rate of radiation is not great at low temperatures, and only becomes equal to the rate of conduction at about 500° C.

$a\gamma$ = the heat capacity of the strip, γ being equal to the product of the thickness of the strip by the specific heat per unit volume.

The factor is analogous to the mass in vibratory mechanics, and the inductance in alternating current calculations.

The equation for T_0, the mean temperature above surroundings is:

$$0.24\left(I_0^2R + \frac{I'^2R}{2}\right) = 2a\beta T_0. \tag{4a}$$

Combining equations (4) and (4a) we have the following, which contains only factors which vary with the time:

$$0.24R\left(2I_0I' \sin pt - \frac{I'^2}{2}\cos 2pt\right) = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt}. \tag{5}$$

In obtaining a solution for $T - T_0$ we shall neglect transient effects, also the double frequency term. The double frequency effect is the principal effect in the case of a pure alternating current supply as given below; but here we simply remark that we can make the double frequency term as small as we please by a suitable choice of the ratio $I_0 : I'$.

The solution of the equation

$$.48RI_0I' \sin pt = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt} \tag{6}$$

is, neglecting transient effects,

$$T - T_0 = \frac{.48RI_0I'}{a\sqrt{4\beta^2 + \gamma^2p^2}} \sin\left(pt - \tan^{-1}\frac{\gamma p}{2\beta}\right), \tag{7}$$

which gives the periodic temperature variation of the strip. Note that if i is the effective (measured) value of the A.C., $\sqrt{2}\,i$ must be written in place of I' in (7).

If the strip is supplied with alternating current only, the fundamental equation becomes

$$0.24RI^2 \sin^2 pt = .12RI^2(1 - \cos 2pt) = 2a\beta T + a\gamma\frac{dT}{dt}. \tag{4'}$$

The mean temperature in this case is defined by

$$.12RI^2 = 2a\beta T_0 \tag{4'a}$$

and the differential equation which $T - T_0$ must satisfy is

$$.12RI^2 \cos 2pt = 2a\beta(T - T_0) + a\gamma\frac{dT}{dt}. \tag{5'}$$

The solution of this equation is, neglecting transient effects

$$T - T_0 = \frac{.12RI^2}{2a\sqrt{\beta^2 + \gamma^2p^2}} \cos\left(2pt - \tan^{-1}\frac{\gamma p}{2\beta}\right). \tag{7'}$$

Having found the magnitude of the temperature variation in the strip, we go on to calculate the magnitude of the effect in the surrounding medium.

Theory of the Effect in the Medium.

Consider an infinite plane metal plate with a column of gas extending normally from either face of a certain portion of the plate; this is equivalent, mechanically, to the strip conductor if terminal conditions are neglected. If the temperature of the plate is a sine function of the time, temperature waves will be propagated into the atmosphere on either side; and calculation will show that these waves are so heavily damped that they are practically extinguished after one wave-length has been traversed. Within this region there is a rise and fall of temperature of the medium with every cycle, and the resulting expansion and contraction of this narrow film of the medium near the source accounts for the sound vibration produced.

In the derivation of equations (7) and (7′) it has been tacitly assumed that no electrical energy was spent in expanding the strip, as this effect would be relatively very small. It is evident from conditions of symmetry that there is no force on the strip tending to make it vibrate; hence no energy can be used mechanically. In calculating the effect on the medium we shall consider two cases:

1. In which the periodic rise in temperature of the strip is allowed to produce a continuous stream of sound energy, propagated away from the strip as plane waves. It is an easy matter to modify this treatment to fit the case of diverging waves in the open atmosphere.

2. In which the strip is placed in a small cavity for the purpose of producing pressure changes; these pressure changes being used to actuate the ear, or some mechanical phonometer which constitutes one wall of the enclosure.

The reason for giving separate treatment to these two types of action, is that in the first case we can speak of a definite amplitude and particle velocity, and a corresponding propagation of energy; whereas in the second case, amplitude and velocity are indefinite terms, and pressure change is much more readily calculated. It is by virtue of pressure change that the acoustic energy generated makes its effect on the bounding wall, and if the dimensions of the cavity are small compared to the acoustic wave-length, the pressure change produced at the strip is quickly distributed over the whole enclosure.

First Case: Wave Propagation from the Strip.—Assume that the periodic temperature variation in the strip results in the expansion and contraction against constant atmospheric pressure of a certain layer of air next to the source. This implies that the very small pressure changes that do arise at the boundary of the layer (as the result of rapid change in volume) are propagated into the atmosphere with such high velocity

that they do not react appreciably on the expansion of the layer. This condition is realized in practice because the velocity of sound in air is so much greater than the velocity of the vibrating boundary which produces the sound.

In treatises on the conduction of heat it is shown that the temperature at any point of the medium distant $\pm x$ from a plane source of temperature, varying periodically as in equation (7), may be expressed as the following function of space and time:

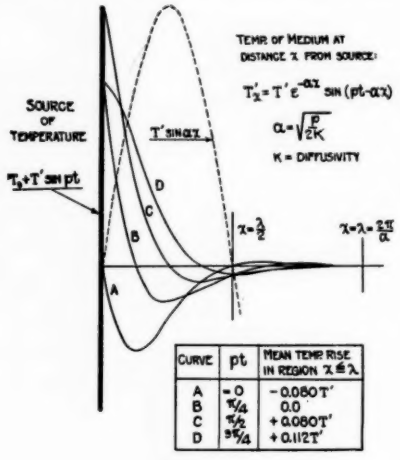
$$T_x' = T'\epsilon^{-\alpha x} \sin (pt \pm \alpha x), \tag{8}$$

in which $\alpha = \sqrt{p/2k}$, p being $2\pi \times$ frequency, and k the "diffusivity" of the medium, or the ratio of the thermal conductivity to the specific heat per unit volume. The value of this constant for air at 0° centigrade, using the specific heat at constant pressure is 0.17 C.G.S. units.

It is necessary to know the effect of the temperature of the medium on k and this can be found by considering separately the conductivity and the specific heat. The former is proportional to the square root of the absolute temperature; the specific heat per unit mass is practically independent of temperature thus making the specific heat per unit volume proportional to the reciprocal of the absolute temperature. Since k is the ratio, we may write

$$k = 0.17 \left(\frac{\theta}{273}\right)^{3/2} \tag{9}$$

where θ denotes the absolute temperature of the medium.



CURVE	pt	MEAN TEMP RISE IN REGION $x \doteq \lambda$
A	0	$-0.080T'$
B	$\pi/4$	0.0
C	$\pi/2$	$+0.080T'$
D	$3\pi/4$	$+0.112T'$

Fig. 2.

The velocity of propagation of the temperature wave is, from (8)

$$v = \frac{p}{\alpha} = \sqrt{2pk}$$

and the wave-length

$$\lambda = \frac{2\pi}{\alpha} = \sqrt{\frac{8\pi^2 k}{p}}. \tag{10}$$

If the wave-length is taken as a unit, it is easy to plot the course of T_x' as a function of x for any given time, t, as is shown in Fig. 2. This shows clearly the enormous damping of these waves of acoustic frequency; it also shows that practically all of the expansion effect due to periodic rise in temperature takes place within the region bounded by the plane

$$x = \lambda = \frac{2\pi}{\alpha}.$$

In order to compute the amount of the periodic expansion, we desire to know the mean value of the temperature rise in this region as a function of the time: that is,

$$\frac{T'}{\lambda}\int_0^{\lambda} \epsilon^{-\alpha x}\sin(pt-\alpha x)dx$$

$$= \frac{T'(1+\epsilon^{-2\pi})}{2\lambda\alpha}(\sin pt - \cos pt) = \frac{T'}{2\sqrt{2\pi}}\sin\left(pt-\frac{\pi}{4}\right) \quad (11)$$

i. e., its maximum value is $.112T'$, and it lags the varying temperature of the strip by the angle $\pi/4$.

If the mean absolute temperature of the air film is θ, the maximum expansion will be,

$$\frac{d\theta}{\theta} = .112\frac{T'}{\theta} \text{ per unit volume,} \quad (12)$$

or per unit length, if expansion is considered to take place in only one direction. The length in question is a wave-length, and this by equations (9) and (10) is, at $\theta°$

$$\lambda = 2\pi\sqrt{\frac{2k}{p}} = 0.82\sqrt{\frac{\pi}{f}}\left(\frac{\theta}{273}\right)^{3/4}. \quad (13)$$

Multiplying (12) and (13) we obtain for the absolute increase in length due to expansion

$$\xi = \frac{.16T'}{\sqrt{f}\,\theta^{\frac{1}{4}}(273)^{\frac{3}{4}}}. \quad (14)$$

This may be considered as the maximum "amplitude" of a sound wave leaving the plane $x = \lambda$, if the effect of the expanding and contracting air film on the surrounding air is the same as that of a solid moving piston—assuming also that the amplitude of the sound produced by a moving piston is equal to the amplitude of the motion of the piston itself.

If the thermal conductivity were proportional to the first power of the absolute temperature, instead of to the square root, we should have had, instead of (12)

$$\xi_1 = \frac{0.16T'}{\sqrt{f}\,.273}. \quad (14a)$$

The departure of (14*a*) from (14) is not serious, if the temperature of the film is 300° C. or less, as in this case $\theta^{1/4}\cdot(273)^{3/4}$ is less than 330. The air film is always considerably cooler than the strip, so that the strip might have a temperature of (say) 500° without causing more than a 20 per cent. discrepancy between ξ and ξ_1.

In order to have the amplitude of the sound wave in terms of the alternating current supplied to the strip, we make use of equations (7) and (7′) which give the variation in temperature of the strip.

Using (7) and (14*a*), we have for a strip supplied with direct current I_0 and alternating current of effective value i, the acoustic amplitude

$$\xi = \frac{4.0 \times 10^{-4} R I_0 i}{a\sqrt{f}\sqrt{4\beta^2 + \gamma^2 p^2}} \sin\left(pt - \tan^{-1}\frac{\gamma p}{2\beta} - \frac{\pi}{4}\right). \tag{15}$$

Using (7′) and (14*a*) we have for a strip supplied with alternating current only

$$\xi' = \frac{7.0 \times 10^{-5} R i^2}{a\sqrt{f}.\sqrt{\beta^2 + \gamma^2 p^2}} \cos\left(2pt - \tan^{-1}\frac{\gamma P}{\beta} - \frac{\pi}{4}\right). \tag{15′}$$

These two equations contain no transient terms; they are solutions for the state of steadily maintained vibrations. The acoustic amplitude ξ' (Equation 15′) is of double the frequency of the applied alternating current.

Using either method of actuating the strip, there is a low critical frequency above which the factor γp (which represents thermal inertia) is so much greater than β (which represents conduction loss, or dissipation) that the latter can be neglected. (This frequency is in the neighborhood of 100 for platinum strip 1 micron thick.) Neglecting β, (15) can be written

$$\xi = \frac{6.4 \times 10^{-5} R I_0 i}{a\gamma f^{3/2}} \sin\left(pt - \frac{3\pi}{4}\right), \tag{15a}$$

and instead of (15′) we have

$$\xi' = \frac{1.1 \times 10^{-5} R i^2}{a\gamma f^{3/2}} \cos\left(2pt - \frac{3\pi}{4}\right). \tag{15′a}$$

In considering how the efficiency of the process depends on the constants of the strip, we note that it is advantageous to make the resistance R as large as possible, and the heat capacity $a\gamma$ as small as possible. The advantage of thinness is plain.

In calculating the intensity of a sound wave, or the rate of flow of energy in the medium it is necessary to know the square of the particle velocity; and this is, from (15*a*) (using superimposed direct current)

$$\dot{\xi}^2_{\max} = p\xi_{\max} = \frac{4.0 \times 10^{-9} R^2 I_0^2 i^4}{a^2\gamma^2 f^3}. \tag{16}$$

Similarly from (15′*a*), for alternating current,

$$\dot{\xi}'^2_{\max} = \frac{1.2 \times 10^{-10} R^2 i^4}{a^2\gamma^2 f^3}. \tag{16′}$$

These equations enable us to find the strength of the source; and knowing this, we can calculate the intensity of the sound at any distance from the source, in the ideal case in which energy is propagated in the form of spherical waves in a homogeneous medium.[1]

Since the dimensions of the source are small compared with the wavelength of sound, we may consider the strip as equivalent to a small sphere of the same area ($2a$) and which produces the same fluid velocity $\dot{\xi}$ at the surface. The velocity potential for the resulting spherical distribution of sound waves is

$$\varphi = -\frac{2a\dot{\xi}_{max}}{4\pi r}\cos\left(pt - \frac{2\pi r}{\lambda}\right), \tag{17}$$

in which $2a\dot{\xi}_{max}$ is the strength of the source, or maximum rate of emission of fluid at the source. In order to calculate the intensity of the sound produced, we make use of the two following equations

$$\text{Intensity} = \frac{W}{t} = \frac{\Pi^2_{max}}{2\rho_0 c}, \tag{18}$$

$$\Pi = -\rho_0\frac{d\varphi}{dt}, \tag{19}$$

in which Π is the pressure change at any point in the field, c the velocity of sound, and ρ_0 the mean density of the medium. Substituting (17) in (19) we obtain for Π in terms of $\dot{\xi}_{max}$

$$\Pi = \frac{-\rho_0 a\dot{\xi}_{max}\cdot 2\pi f}{2\pi r}\sin\left(pt - \frac{2\pi r}{\lambda}\right),$$

and for the intensity, according to equation (18)

$$\frac{W}{t} = \frac{\rho_0 a^2\dot{\xi}^2_{max}\cdot f^2}{2cr^2}, \tag{20}$$

or finally in terms of the electrical energy used in the strip (direct current case)

$$\frac{W}{t} = \frac{2\times 10^{-9}R^2I_0^2i^2\rho_0 f}{cr^2\gamma^2}, \tag{21}$$

or, for alternating current,

$$\frac{W}{t} = \frac{6.0\times 10^{-11}R^2i^4\rho_0 f}{cr^2\gamma^2}. \tag{21'}$$

[1] The solution here given for intensity in the case of ideal spherical distribution may easily be applied to the more practical case in which the small thermophone element is placed close to an infinite rigid plane wall. In this case, the velocity potential on the thermophone side of the wall will be twice as great as given by (17) and the intensity four times as great as given in (20).

Thus the actual intensity at any point some distance away from a thermophone whose power input is constant should increase with the first power of the frequency, and decrease with the square of the distance r. It is independent of a, the area of the strip.

Second Case: Production of Pressure Changes in Small Enclosure.— Let us assume that the strip is placed in an enclosure the dimensions of which are small compared with the acoustic wave-length, and further that the shortest distance from the strip to the boundary is large compared to the wave-length of the heat wave originating at the surface of the strip. These conditions are readily satisfied for all ordinary acoustic frequencies. If the temperature variation of the strip is given by

$$T' \sin \omega t$$

the temperature variation at any near-by point in the enclosure is

$$T_x' = T'\epsilon^{-\alpha x} \sin (\omega t \pm \alpha x). \tag{8}$$

We can consider that both sides of the strip, each of area a, give rise jointly to the temperature wave; also that this temperature wave travels a mean distance x before striking boundary defined by the equation

$$\bar{x} = \frac{V_0}{2a},$$

where V_0 is the volume of the enclosure. The alternating temperature averaged over the whole enclosure is then

$$\delta T = \frac{2a}{V_0}\int_0^{\bar{x}} T_x' dx = \frac{2aT'}{V_0}\int^{\bar{x}} E^{-\alpha x} \sin (\omega t - \alpha x) dx. \tag{22}$$

The thermal conductivity of the gaseous medium varies as the square root of the absolute temperature, while the specific heat per unit volume is practically constant at constant volume, so that the diffusivity is

$$K = K_0\sqrt{\frac{\theta_1}{273}}.$$

In terms of K_0, the diffusivity at o° Centigrade, θ_1 is the absolute temperature of the gas near the element, this being approximately the same as the temperature of the element itself.

We then have

$$\alpha = \sqrt{\frac{\omega}{2K_0}}\sqrt{\frac{273}{\theta_1}}. \tag{23}$$

As α varies only as the fourth root of $\theta_1/273$, and conditions are easily arranged so that the temperature of the gas is not excessive, α may be considered constant in the evaluation of the integral in (17). Integrating,

$$\delta T = \frac{aT'}{V_0\alpha}\left[\epsilon^{-(V_0a/2a)}\left\{\sin\left(\omega t + \frac{V_0\alpha}{2a}\right) - \cos\left(\omega t + \frac{V_0\alpha}{2a}\right)\right\} - \sin\omega t - \cos\omega t\right].$$

Now K_0 is of the order of unity ($K_0 = 1.5$ for hydrogen and .23 for air, using specific heat at constant volume) so that α (equation 23) is large for all acoustic frequencies. We may, therefore, neglect $\epsilon^{-(V_0a/2\alpha)}$ and write

$$\delta T = -\frac{\sqrt{2}\,aT'}{V_0\alpha}\sin\left(\omega t - \frac{\pi}{4}\right)$$

and, substituting the value of α from (23)

$$\delta T = -\frac{2aT'}{V_0\sqrt{\omega}}\sqrt{K\sqrt{\frac{\theta_1}{273}}}\sin\left(\omega t - \frac{\pi}{4}\right). \qquad (24)$$

If the walls of the boundary are rigid, we have for a perfect contained gas, $\delta V = 0$ and the pressure change in terms of temperature change is

$$\Pi = \frac{P}{\theta_2}dT$$

if P = total pressure and θ_2 is the mean temperature of the gas. Substituting δT from (24) we have for pressure change in the enclosure

$$\Pi = \frac{2aT'P\sqrt{K_0\sqrt{\theta_1/273}}}{\theta_2 V_0\sqrt{\omega}}\sin\left(\omega t - \frac{\pi}{4}\right), \qquad (25)$$

in terms of temperature variation in the strip. When direct current is used with the A.C. this is given by (7); substituting this expression for T' and dropping the dissipation factor β, we have, ($\omega = p$)

$$\Pi = \frac{.086RI_0iP\sqrt{K_0\sqrt{\theta_1/273}}}{\gamma V_0\theta_2 f^{3/2}}\sin\left(pt - \frac{3\pi}{4}\right) \qquad (26)$$

and when the strip is actuated only by alternating current, we have from (25) and (7′), dropping β as before, and noting that $\omega = 2p$,

$$\Pi = \frac{.0106Ri^2P\sqrt{K_0\sqrt{\theta_1/273}}}{\gamma V_0\theta_2 f^{3/2}}\cos\left(2pt - \frac{3\pi}{4}\right). \qquad (26')$$

In (26′) f is the frequency of the alternating current and half the acoustic frequency.

Equations (26) and (26′) are in the most convenient form for calculating the stress exerted on any part of the boundary, which may be the exposed face of a sound detecting mechanism, as for example the ear. The intensity of the sound produced in the enclosure can easily be com-

puted from the usual equations

$$\frac{W}{t} = \tfrac{1}{2}\rho_0 c^3 s^2 = \tfrac{1}{2}\frac{\Pi^2}{\rho_0 c}, \tag{27}$$

in which s = maximum condensation (Π/P), Π = maximum pressure change, ρ_0 = mean density, and c = velocity of sound in medium. Substituting the value of Π from (26) in (27), the intensity is, in the case of direct current operation

$$\frac{W}{t} = \frac{3.7 \times 10^{-3} R^2 I_0^2 i^2 P^2 K_0 \sqrt{\theta_1/273}}{\rho_0 c \gamma^2 V_0^2 \theta_2 f^3}, \tag{28}$$

and in the case of alternating current only, from (26′)

$$\frac{W}{t} = \frac{1.1 \times 10^{-4} R^2 i^4 P^2 K_0 \sqrt{\theta_1/273}}{\rho_0 c \gamma^2 V_0^2 \theta_2^2 f^3}. \tag{28'}$$

It is seen from these equations that the intensity in this case is inversely proportional to the cube of the frequency. The temperature θ has been retained in equations (28) and (28′), and the calculation has been carried through to a determination of the intensity; but there is not much difference between equations (21), (21′) which deal with the intensity in the first case, and (28), and (28′) which deal with the intensity in the second case, except the frequency-variation law.

In all cases the temperatures of gas and of strip must be taken into account; and in most cases it is possible to arrange experimental work and calculation so that this can be done in a very simple way.

EXPERIMENTAL TESTS.

The first test that was made was a rough verification of equations (15*a*) and (26) to see if the computed effect was of the right order of magnitude. The method used consisted in setting the thermophone and an electro-mechanical source (ordinary telephone receiver) for equal intensity at the same pitch, and measuring the electrical input into each instrument. The setting for equal intensity was made with the unaided ear, for simple experiments have shown that the ear judges equality between two tones of the same pitch to within 4 or 5 per cent.[1] The telephone receiver had previously been calibrated as a sound generator by measuring the motion of the diaphragm with a microscope when a known value of alternating current was sent through it. In the case of the vibrating telephone diaphragm, the motion of the diaphragm is greatest near the center, falling off to zero at the edge. The law of

[1] Or to one per cent. under favorable conditions. The ear seems to be about as good in these measurements as the eye is in the analogous photometrical case.

distribution of amplitude over the diaphragm is, for small vibrations (at the particular frequency used), such that the bowed diaphragm may be considered from the standpoint of air displacement as replaced by a piston whose area is 0.306 that of the diaphragm, and which moves back and forth with an amplitude equal to the amplitude of the diaphragm at the center.

The data of this experiment were:

Frequency, 800.

Constants of telephone receiver:

Area of diaphragm, 18.3 sq. cm.

Effective area, 5.5 sq. cm.

800-cycle current, 1.7×10^{-5} amp.

Amplitude at center of diaphragm 1.85×10^{-6} cm.

Constants of thermophone element:

Material, platinum, of thickness 7×10^{-5} cm.

Area $a = 0.8$ sq. cm.

Effective area $2a = 1.6$ sq. cm.

γ = (thickness times specific heat per unit volume) $= 5 \times 10^{-5}$

Resistance 1.0 ohm

Direct current $I_0 = 1.2$ amperes.

800-cycle current $= 5.6 \times 10^{-2}$ amp.

The amplitude (ξ_{max}) is computed from (15a) corrected for temperature as per (14):

$$\xi_{max} = \frac{6.4 \times 10^{-5} R I_0 i \cdot \sqrt[4]{273}}{a\gamma f^{3/2}\sqrt{\theta}}. \qquad (15b)$$

Allowing for a temperature of about 150° centigrade ($\theta = 423$), we compute

$$\xi_{max} = 4.2 \times 10^{-6} \text{ cm.}$$

In comparing the acoustic outputs from these two sources, we shall assume that they are two pistons which communicate their amplitudes of motion to the adjacent medium. The strength of each source should be proportional (at fixed frequency) to the area of the piston times the amplitude of its motion. In the case of the telephone receiver, this quantity is $5.5 \times 1.85 \times 10^{-6} = 1.02 \times 10^{-5}$ cm.^{3}; and in the case of the thermophone element, $1.6 \times 4.2 \times 10^{-6} = 0.67 \times 10^{-5}$ cm.^{3} In these experiments the thermophone element was fitted into a receiver case, similar to that of the telephone receiver, and both instruments were held *loosely* to the ear. Assuming them to be *tightly* held it would be more correct to compute, instead of displacement, the relative pressure changes in the enclosed volume of air, (V_0) in order to compare the two

sources. In the case of the telephone receiver the pressure change would be

$$\frac{\Pi}{P} = \frac{1.02 \times 10^{-5}}{V_0}$$

and for the thermophone, using equation (26)

$$\frac{\Pi}{P} = \frac{0.89 \times 10^{-5}}{V_0} .$$

The agreement between the two values, computed in either way is fairly good, considering the number of factors that have to be taken into account in making the comparison.

A second experimental test was made for the purpose of verifying the intensity-frequency relation given in equation (28). Ear comparison of intensities was again resorted to, the energy from the strip conductor being compared with that from a special telephone receiver at various frequencies. (The dynamical characteristics of the telephone receiver had been roughly determined so that it was possible to regulate it for equal acoustic output at various frequencies by adjusting the alternating current input.) The A.C. power input i^2R in the strip was measured for equal intensity at several frequencies, and the results are shown in Fig. 3.

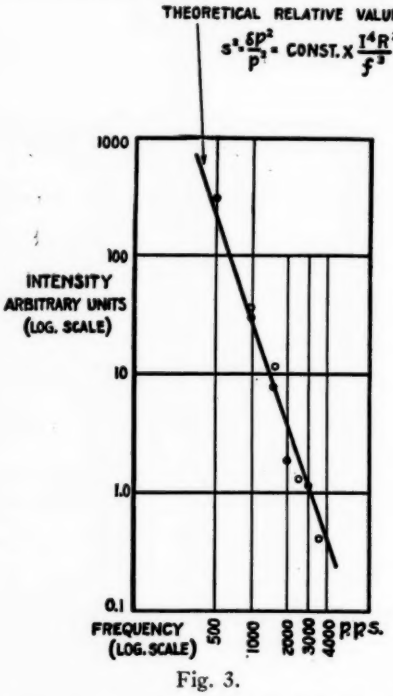


Fig. 3.

Intensity-Frequency Relation in Enclosure.

The points represent the relative intensity at different frequencies for equal A.C. power input, and are proportional to the reciprocal of the power input for equal intensity at each frequency. The curve represents the theoretical decrease in intensity according to the cube of the frequency, and the general result is a confirmation of this relation.

The writers are indebted to Mr. E. C. Wente of this laboratory for an experimental method and data which afford a much more accurate and satisfactory test of the theory than the two experiments given above.[1] The thermophone element was placed in an enclosure whose

[1] This experiment was carried out by Mr. Wente in connection with work on the theory and calibration of a new phonometer which is reported on in the paper immediately following.

volume V_0 was about 45 cubic centimeters; one of the walls of which consisted in a phonometer or pressure-measuring instrument as shown in Fig. 4*a*. This wall yielded so little that the experiment can be considered as carried out rigorously under constant volume. The pressure change in this case, if only alternating current is used to actuate the strip, is given by equation (26′). The experiments were made at a frequency of 20 cycles, the (platinum) strip being made sufficiently heavy to give a large value of thermal inertia γp so that the dissipation term β could be neglected. In order to eliminate an absolute calibration of the phonometer, a second experiment was made, using the piston apparatus shown in Fig. 4*b*, at the same frequency. The maximum pressure change Π as produced by the piston is easily calculated from mechanical considerations, and the comparison is easily made.

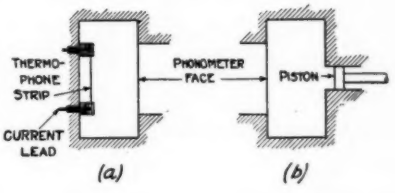


Fig. 4*a*. Fig. 4*b*.

When the piston apparatus was used, the ratio of phonometer reading to calculated pressure increase was 2.02 arbitrary units; and when the strip conductor was used, the ratio of the phonometer reading to pressure change as calculated from (26′) was 1.92 on the same scale. This confirmation to within 5 per cent., was the best we have had of the theory given in this paper.

The results obtained with platinum show that good quantitative work can be done with the thermophone when this material is used for the element. However, it is possible to obtain other materials, such as gold leaf, which are much thinner than bolometer platinum—and which are therefore very useful in cases where higher efficiency is needed. Caution should be used in applying the theoretical formulæ to elements of gold leaf since the heat capacity of gold leaf seems to be very different in different samples. Any such variations, due perhaps to absorbed gases, may be cared for (as shown by E. C. Wente in the following paper) if a check can be made against a platinum element in the same atmosphere. The correction factor thus obtained should hold for all frequencies so long as the gold foil is not unduly heated.

COMPARATIVE VALUE OF THE THERMOPHONE AS A LABORATORY SOURCE OF SOUND.

With regard to efficiency the thermophone compares favorably with electromagnetic and electrostatic devices except in the vicinity of their natural frequencies. In certain work it is essential that the response

should be as nearly uniform as possible over a wide range of frequencies and that the relative response should be easily determinable. For such work the advantages of the thermophone are evident, for while its response diminishes with increasing frequency the law of variation is simple. When sound of indeterminate loudness and of one frequency only is desired the volume obtainable from the thermophone does not compare favorably with that from resonant mechanical devices.

The thermophone is particularly adapted to laboratory purposes because it requires no adjustment. It is extremely simple in structure and the units are readily reproducible. The determination of the acoustic effect of the thermophone depends principally upon the thermal properties of materials and is remarkably simple as compared with corresponding determinations for resonant apparatus, which usually involve motions of complicated mechanical systems. In addition, the response of the thermophone is uniform through indefinite periods of time and is not subject to the trouble of accidental detuning, which so often occurs in resonant apparatus.

Possibly even more important than the ease of determination of the sound effects in the air close to the element is the fact that these sound effects cannot react appreciably upon the source of energy whence they arise. Whenever a vibratory system is used it is always subject to reactions which may present serious complications. The thermophone seems the nearest equivalent to an ideal piston source at present obtainable.

Various modifications of size, shape and electrical resistance of the thin conductor employed may be necessary in experimental work. These need change the theory given in no essential way. On account of its simplicity from theoretical and practical points of view we believe that the thermophone in conjunction with a suitable supply of alternating current will be of material value as a precision source of sound.

SUMMARY.

1. A description of a simple thermophone structure is given together with the theory of its operation.

2. An account is given of experimental tests the results of which are substantially in accord with the theory.

3. The thermophone is adapted to two classes of service (*a*) as a precision source of sound at any frequency (*b*) as a source of sound of known relative loudness at different frequencies throughout the acoustic range.

RESEARCH LABORATORY OF THE AMERICAN
TELEPHONE AND TELEGRAPH CO. AND WESTERN ELECTRIC COMPANY, INC.

A CONDENSER TRANSMITTER AS A UNIFORMLY SENSITIVE INSTRUMENT FOR THE ABSOLUTE MEASUREMENT OF SOUND INTENSITY.

BY E. C. WENTE.

THE various methods that have been used with more or less success for measuring the intensity of sound may be divided into five general classes: observation of the variation in index of refraction of the air by an optical interference method; measurement of the static pressure exerted on a reflecting wall; the use of a Rayleigh disc with a resonator; methods in which the motion of a diaphragm is observed by an optical method; the use of some type of telephone transmitter in connection with auxiliary electrical apparatus. The apparatus of either of the first two methods is non-resonant and hence the sensitiveness is fairly uniform over a wide range of frequencies. These methods are not sufficiently sensitive, however, to be of use in general acoustic measurements. On the other hand, instruments of the last three classes possess a natural frequency and are consequently very efficient in the resonance region. However, in the neighborhood of the resonant frequency the efficiency varies greatly with the pitch of the tone. It is possible to use a Rayleigh disc without a resonator, but its sensitiveness in that case is so low that it is of little practical value.

Because of the recent advances in the development of distortionless current amplifiers, the last class, in which use is made of some form of telephone transmitter, seems to offer the greatest possibilities. In the following pages a transmitter is described which has been calibrated in absolute terms for frequencies from 0 up to 10,000 periods per second and which has a nearly uniform sensibility over this range. The apparatus is easily portable, and possesses no delicate parts, so that, when once adjusted, it will remain so for a long period of time.

Except in cases where measurements are made with a single, continuous tone, it is desirable that the instrument for measuring the intensity of sound should have approximately the same sensibility over the entire range of frequencies used. This is especially important if the sound under investigation has a complex wave form. To avoid any great variation with frequency in the sensibility of a phonometer employing a vibrating

system, it is necessary that the natural frequency lie outside the range of frequencies of the tones to be measured. Even if the natural frequency be compensated for in other ways, small variations in the constants of the instruments, which are always likely to occur, may change conditions appreciably at this frequency. It is pretty well recognized that for several reasons the natural frequency should lie above rather than below the acoustic range. If the instrument is to be used in studying speech, the natural frequency must indeed be very high. The upper limit of the frequencies occurring in speech is not definitely known, but it probably does not come below 8,000 periods a second. Titchener[1] found that if a Galton whistle was set so as to give a frequency of 8,500, the tone emitted could not be distinguished from an ordinary hiss.

An instrument that is to be used in studying speech should have high damping as well as a high natural frequency in order to reduce distortion due to transients. This is not so important if the natural frequency lies beyond the acoustic range, but nevertheless is desirable even in this case. Aperiodic damping is the best condition, but it is in general hard to obtain when the natural frequency is very high.

It seems best in this paper to give a rather complete treatment of the condenser instrument; for the sake of clearness, however, breaking up the matter into a number of sections as follows:

1. Theory of the Operation of an Electrostatic Transmitter.
2. General Features of the Design of the Instrument.
3. Deflection of the Diaphragm under a Static Force. Measurement of Tension and Airgap.
4. Sensitiveness of the Transmitter at Low Frequencies.
5. Sensitiveness at Higher Frequencies Determined by the Use of a Thermophone.
6. Natural Frequency and Damping of the Diaphragm.
7. Possibilities of Tuning.
8. Characteristic Features of the Instrument.
9. The Electrostatic Instrument used as a Standard Source of Sound.
10. Summary.

Some of these sections deal with theory and some with experimental work as need arises, the general aim being to put in proper order the material necessary for a full account of the condenser instrument.

1. Theory of the Operation of an Electro-static Transmitter.

The device to be described is a condenser transmitter, the capacity of which follows very closely the pressure variations in the sound waves. The use of such a device as a transmitter is not a new idea; in fact it

[1] Proc. Am. Phil. Soc., 53, p. 323.

was suggested almost as early as that of the corresponding electromagnetic instrument.[1] However, before good current amplifiers were available little or no use was made of electrostatic transmitters because of their comparatively low efficiency.

A simple circuit that may be used with such a transmitter is shown in Fig. 1. When the capacity of the transmitter is varied, there will be a corresponding drop of potential across R, which may be measured with an A.C. voltmeter or some other suitable device.

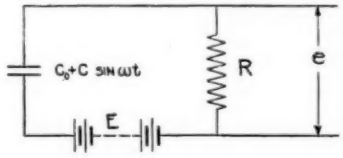


Fig. 1.

In order to get a quantitative expression for the magnitude of this voltage let us assume that the capacity at any instant is given by

$$C = C_0 + C_1 \sin \omega t,$$

in which $\omega = 2\pi \times$ frequency. For the circuit shown in Fig. 1

$$E - Ri = \frac{1}{C} \int i dt. \tag{1}$$

By differentiation and substitution we obtain

$$(C_0 + C_1 \sin \omega t) R \frac{di}{dt} + (1 + RC_1\omega \cos \omega t) i - EC_1\omega \cos \omega t = 0. \tag{2}$$

In order to evaluate this equation let us assume as a solution

$$i = \Sigma I_n \sin (n\omega t + \phi_n).$$

Substituting this value of i in (2) and determining the coefficients, we have

$$\left.\begin{aligned} i = {} & \frac{EC_1}{C_0 \sqrt{\left(\frac{1}{C_0\omega}\right)^2 + R^2}} (\sin \omega t + \varphi_1) \\ & - \frac{EC_1^2 R}{C_0^2 \sqrt{\left[\left(\frac{1}{C_0\omega}\right)^2 + 4R^2\right]\left[\left(\frac{1}{C_0\omega}\right)^2 + R^2\right]}} \sin (2\omega t + \varphi_1 - \varphi_2) \\ & + \text{terms of higher order in } C_1/C_0, \end{aligned}\right\} \tag{3}$$

in which

$$\phi_1 = \tan^{-1} \frac{1}{C_0\omega R} \quad \text{and} \quad \varphi_2 = \tan^{-1} \frac{1}{2C_0\omega R}, \quad \text{etc.}$$

[1] La Lumiere Electrique, Vol. 3, p. 286, 1881.

For the best efficiency R should be made large in comparison with $1/C_0\omega$. In this case, the expression for the voltage e becomes

$$e = Ri = \frac{EC_1}{C_0} \sin(\omega t + \varphi_1) - \frac{EC_1^2}{2C_0^2} \sin(2\omega t + \varphi_1 - \varphi_2) + \cdots .$$

From this equation we see that in order to get a voltage of pure sine wave form for a harmonic variation of capacity, C_1 must be small in comparison with $2C_0$. This condition is satisfied as long as the A.C. voltage is small compared with E.

Retaining only the first term in (3) we have

$$e = Ri = \frac{EC_1R}{C_0\sqrt{\frac{1}{C_0^2\omega^2} + R^2}} \sin(\omega t + \varphi_1). \tag{4}$$

This equation shows that, so far as its operation in the circuit is concerned, the transmitter may be considered an alternating current generator giving an open circuit voltage $E(C_1/C_0) \sin(\omega t + \varphi_1)$ and having an internal impedance $1/C_0\omega$. It can also be shown that the transmitter can be regarded from this point of view if R is replaced by a leaky condenser or an inductance, so that this result may be said to be true in general.

2. General Features in the Design of the Instrument.

The general construction of the transmitter is shown in Fig. 2, from which the principal features are evident. The diaphragm is made of steel, 0.007 cm. in thickness, and is stretched nearly to its elastic limit. The condenser is formed by the plate B and the diaphragm. Since the diaphragm motion is greatest near the center, the voltage generated, which is proportional to C_1/C_0, will be greatest if the plate is small. On the other hand, since C_0 is proportional to the size of the plate, it cannot be made too small or the internal impedance of the transmitter will be too great. Therefore from the standpoint of efficiency, a compromise has to be made in determining the area of the plate. However, if it is made much smaller than the diaphragm, the natural frequency of the vibrating system will be decreased, as is explained below. On the basis of these factors the size of the plate indicated was judged to be about the best for the transmitter.

After some experiments with various dielectrics between the plate and the diaphragm it was concluded that air was most suitable. The dielectric constant of air is not so high as that of some other materials, but its insulating properties are better. However, the principal advantage of using air is, that it has a high minimum value of sparking

potential which lies in the neighborhood of 400 volts, below which there is no appreciable conduction. When E is less than this voltage, the air gap may be decreased without decreasing E, so that the efficiency of the instrument is limited practically only by the fact that when the gap is decreased below a certain value, the electrostatic force between the plate and diaphragm deflects the latter sufficiently to short circuit the condenser. When a potential difference of 320 volts was applied to the transmitter shown in Fig. 2, no appreciable current flowed across the air gap, certainly not more than 10^{-8} amperes. The fact that the air has such a high minimum sparking potential is one of the principal reasons why it is possible to design a successful condenser transmitter of the type shown in Fig. 2.

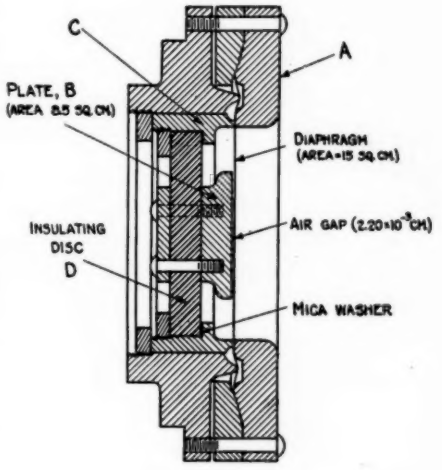


Fig. 2.

Sectional drawing of transmitter.

A word may be said in regard to the method of adjusting the transmitter so as to obtain a small uniform air gap. The surface of part A, next to the diaphragm, was ground plane before assembling. Small irregularities in the surface of the diaphragm facing the plate were removed by grinding with fine carborundum. Parts B, C and D were first assembled without the mica washer. The face of the plate and the ends of part C were then ground to the same level. Finally the mica washer was inserted between C and D and the whole apparatus assembled as shown. The mica may be split into washers of very even thickness, and a uniform air gap so obtained. The diaphragm is clamped between parts A and C, and is thus held in a true plane. In assembling the parts, the greatest care must be taken that no dust is caught between the plate and the diaphragm, for the insulation may be considerably reduced by the presence of any small particles in the gap.

Part C does not fit so perfectly against the diaphragm that the space surrounding the plate is shut off completely from the outside air. Changes in temperature and atmospheric pressure will therefore not affect the equilibrium position of the diaphragm.

The instrument used in these experiments was constructed just as

shown in Fig. 2. It is evident from this figure that the diaphragm may be brought into contact with the plate if a mechanical pressure is accidentally exerted on the diaphragm. This will cause a spark to pass, if the transmitter was previously charged. In order to avoid damaging the metal surfaces in this way it may be advisable to glue to the face of the plate, A, a very thin layer of mica of uniform thickness, while still retaining an air-gap sufficient to allow free motion of the diaphragm.

3. Deflection of the diaphragm under a Static Force; Measurement of Tension and Air Gap.

It is not difficult to calculate the sensitiveness of the transmitter for low frequencies from the dimensions of its various parts, provided the magnitude of the deflection of every point of the diaphragm produced by a given static force is known. Since the diaphragm is made of very thin material and the tension is high, we may expect the diaphragm to behave very much as an ideal membrane, at least for frequencies near zero. In order to determine how closely this condition is approximated the following experiment was carried out.

When a static potential is applied between the plate and the diaphragm, the latter is deflected by the electrostatic force. The deflection produced

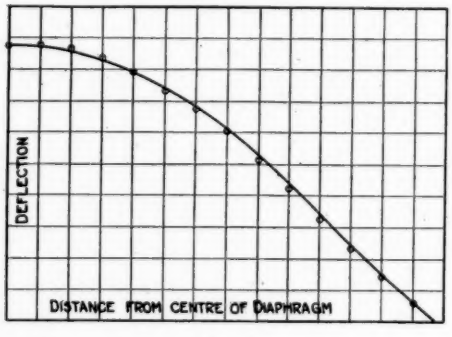


Fig. 3.

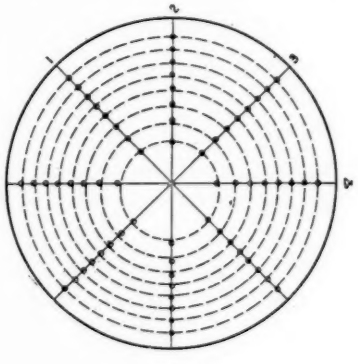


Fig. 4.

in this way by a known potential was measured by a device very similar to that used by Prof. D. C. Miller in his phonodeik.[1] By this arrangement the deflection of the diaphragm was magnified 30,000 times. The mean values of the deflections produced at various points along eight evenly spaced radii when a potential of 320 volts was applied are shown in Fig. 3. Points of equal displacement of the diaphragm are plotted in Fig. 4. The fact that the curves drawn through these points are

[1] D. C. Miller, Science of Musical Sounds, p. 79.

practically circles shows that the tension of the diaphragm was very nearly the same in all directions.

The distance between the plate and diaphragm was also measured with this apparatus by applying a mechanical force until the diaphragm touched the plate. The value obtained in this way was 2.20×10^{-3} cm. The capacity of the transmitter was measured on a capacity bridge and found to be 335×10^{-12} farads, from which the computed value of the air gap is 2.25×10^{-3} cm. The mean of the values obtained in these two ways is 2.22×10^{-3} cms.

In order to determine how closely the diaphragm approximates an ideal membrane, we may calculate the form that the latter would have assumed under the conditions of the preceding experiment.

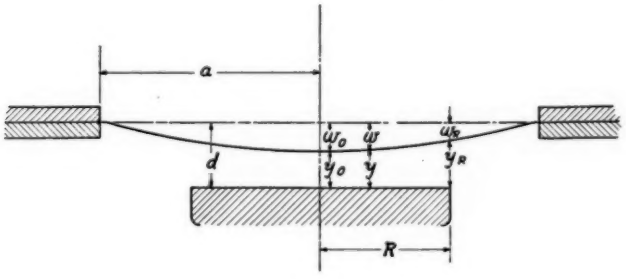


Fig. 5.

Referring to Fig. 5, if V is the potential between the plate and the diaphragm, and T, the tension of the membrane, we have

$$T\left[\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right] + \frac{V^2}{8\pi y^2} = 0.^1$$

This relation holds from $r = 0$ to $r = R$. Let

$$A = \frac{V^2}{8\pi T}$$

and $x = \log r$, then since $(w - w_R) = (Y_R - y)$

$$\frac{d^2y}{dx^2} = A\frac{\epsilon^{2x}}{y},$$

or, since $(w - w_R)/(w_0 - w_R)$ is very nearly equal to $(R^2 - r^2)/r^2$ and $(w_0 - w_R)$ is small compared with y_0,

$$\frac{d^2y}{dx^2} = \frac{A}{y_0}\left(1 - 2k\frac{r^2}{R^2}\right)\epsilon^{2x},$$

in which

$$k = (w_0 - w_R)/y_0.$$

[1] Rayleigh, Theory of Sound, II., p. 318.

From this we get

$$w = \frac{A}{4y_0^2}(R^2 - r^2)\left[1 - \frac{k}{2} - \frac{k}{2}\frac{r^2}{R^2}\right] + w_R. \tag{5}$$

The total force on the diaphragm is

$$F = \frac{\pi R^2 V^2}{8\pi d_1^2}$$

where d_1 is defined by the equation

$$\frac{\pi R^2}{d_1} = \int_0^R \frac{2\pi r dr}{y^2} = \frac{2\pi}{y_0}\int_0^R \left(1 - 2k\frac{r^2}{R^2}\right) r dr = \frac{\pi R^2}{y_0}(1 - k).$$

so that

$$F = \frac{\pi R^2 V^2}{8\pi y_0^2}(1 - k).$$

In the region extending from $r = R$ to $r = a$,

$$F = -2\pi r T \frac{dw}{dr}.$$

From this

$$w_R = \frac{AR^2(1 - k)}{2y_0}\log\frac{a}{R}. \tag{6}$$

From (5) and (6)

$$w = \frac{A}{4y_0^2}\left\{(R^2 - r^2)\left[1 - \frac{k}{2}\left(1 + \frac{r^2}{R^2}\right)\right] + 2R^2(1 - k)\log\frac{a}{R}\right\}. \tag{7}$$

This equation gives the form into which the diaphragm will be bent if it behaves like an ideal membrane. The curve representing this equation is shown in Fig. 3. The observed points do not lie very far from this curve. We therefore conclude that the diaphragm behaves sufficiently like an ideal membrane, so that no great error will be incurred if this assumption is made in calculating the sensitiveness of the transmitter for low frequencies.

From equation (7)

$$w_0 = \frac{V^2}{32\pi T y_0^2}\left\{R^2\left(1 - \frac{k}{2}\right) + 2R^2(1 - k)\log\frac{a}{R}\right\}$$

or

$$T = \frac{V^2 R^2}{32\pi y_0^2 w_0}\left\{\left(1 - \frac{k}{2}\right) + 2(1 - k)\log\frac{a}{R}\right\}. \tag{8}$$

Hence, if the deflection at the center of the diaphragm produced by a known voltage is measured, the tension may be calculated from (8). Results obtained in this way for the diaphragm used in these experiments are tabulated below.

Volts.	Deflection (w_0) (cm.).	Tension (T) $\frac{\text{(dynes)}}{\text{(cm.)}}$.
200	6.0×10^{-5}	6.59×10^{7}
240	6.8	6.58
280	12.4	6.55
320	16.9	6.55
Mean		6.57×10^{7}

4. Sensitiveness of the Transmitter at Low Frequencies.

Having satisfied ourselves that the diaphragm behaves sufficiently like a perfect membrane, and having determined the tension and air gap, we can now proceed to calculate the efficiency of the transmitter for low frequencies. To do this it is necessary to find the change in capacity produced by a given pressure on the diaphragm, since by equation (4) the voltage generated is proportional to C_1/C_0.

Referring to Fig. 5, we see that the capacity is $R^2/4d$ if the diaphragm is not deflected. From the curve of deformation when a potential is applied (Fig. 3), it is evident that w_R is very nearly equal to 0.45 w_0. Hence the air gap at any point is given by

$$d - w_0 + \frac{.55}{R^2} w_0 r^2,$$

and since the surface of the diaphragm deviates but little from a plane area, the normal capacity to the first approximation is

$$C_0 = \int_0^R \frac{2\pi r dr}{4\pi\left(d - w_0 + \frac{0.55 w_0}{R^2} r^2\right)} = \frac{R^2}{4d^2}\left[1 + \frac{w_0}{d} - \frac{.55 w_0}{2d}\right] = \frac{R^2}{4d'}, \quad (9)$$

in which d' may be called the effective air gap.

If a pressure, P, uniform all over the diaphragm produces a deflection, u, the capacity of the condenser will have been changed by the amount

$$C_1 = \int_0^R \frac{u \cdot 2\pi r dr}{4\pi y^2}. \quad (10)$$

The quantity in brackets of equation (9) does not differ greatly from unity in any practical case, so that no great error will be incurred if we set y in (10) equal to the constant value d'. Since

$$u = \frac{P}{4T}(a^2 - r^2),^1$$

equation (16) may be written

$$C_1 = \frac{P}{8Td'^2}\int_0^R (a^2 - r^2) r dr = \frac{PR^2}{32Td'^2}[2a^2 - R^2]. \quad (11)$$

[1] Lamb, Dynamical Theory of Sound, p. 150.

C_1 is the change in capacity produced by a static pressure, P; but this differs very little from the maximum value of the alternating capacity resulting from a pressure, $P \sin \omega t$, provided $\omega/2\pi$ is small compared with the natural frequency of the diaphragm.

Having determined C_1 per unit value of P from equation (11), and C_0 from (9), we may calculate C_1/C_0 and hence the sensitiveness, *i. e.*, the volts per unit pressure. In practically all the experiments that have been made with the electrostatic transmitter, the D.C. voltage was 321. Under this condition we obtain 315 E.S.U. for C_0 from (9) and 1.96×10^{-3} E.S.U. per dyne per sq. cm. for C_1/P from (11). Hence we have for the sensitiveness

$$\frac{EC_1}{PC_0} = \frac{1.96 \times 10^{-3} \times 321}{315} = \frac{2.00 \times 10^{-3} \text{ volts}}{\text{dynes per sq. cm.}}.$$

In order to check this value directly by experiment, the apparatus diagrammatically shown in Fig. 6 was constructed. A receptacle was

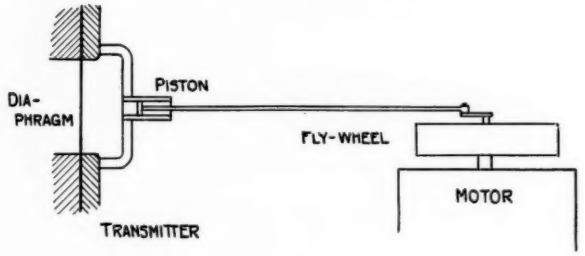


Fig. 6.

placed over the diaphragm as shown in the figure, thus forming an air-tight enclosure. Connected to this was a cylinder containing a piston. The connecting rod was long compared with the stroke of the piston so that with the motor running, the piston was given practically a simple harmonic motion. The fly wheel was fairly heavy and the connecting rod was made of stiff tubing, so that but little vibration was noticeable even when the motor ran at the highest speed.

The pressure variation is given by

$$\delta P = 1.4P\frac{\delta V}{V},$$

in which δV is one half the total piston displacement and P is the maximum value of the alternating pressure.

$$V = 45.2 \text{ c.c. (volume of chamber)}$$

$$\delta V = \frac{0.68 \times 0.418}{2} = .142 \text{ c.c.}$$

Hence

$$\delta P = 1.4 \times 10^6 \times \frac{1.42}{45.2} = 4{,}400 \text{ dynes per cm.}^2$$

The root mean square value of the pressure is

$$\frac{4{,}400}{\sqrt{2}} = 3{,}120 \text{ dynes per cm.}^2.$$

The circuit used in this test is shown in Fig. 7. The electrostatic voltmeter had a very small capacity, giving it at low frequency an impedance large compared with the 80 megohm resistance in shunt.

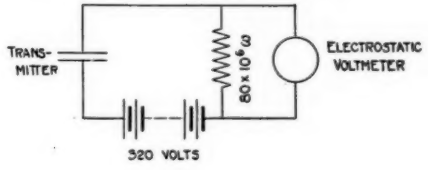


Fig. 7.

We may then calculate the open circuit voltage given by the transmitter from the voltmeter reading and the constants of the circuit, remembering that the transmitter may be regarded as a generator having an internal impedance $1/C_0\omega$. The following values were obtained in this way.

Motor Speed, R.P.M.	Frequency (P.P.S.).	Voltmeter Reading (Volts).	Open Circuit Volts.
1239	20.7	5.31	6.22
1074	17.9	5.31	6.27
950	15.8	5.31	6.33
824	13.75	5.20	6.29
584	9.75	4.92	6.27
Mean.....	.....	.....	.....6.28

We therefore have for the sensitiveness,

$$\frac{6.28}{3120} = 2.02 \times 10^{-3} \frac{\text{Volts}}{\text{dynes per sq. cm.}}.$$

This value is in very close agreement with that given before, so that we may consider 2.00×10^{-3} volts per dyne as a reasonably correct value for the sensitiveness at low frequencies.

5. Sensitiveness at Higher Frequencies as Determined by the Use of a Thermophone.

By the methods just described the values of sensitiveness may be determined for very low frequencies only. In order to measure the

sensitiveness at higher frequencies and also to get an idea of the natural frequency and damping of the vibrating system, use was made of the principle involved in the action of the thermophone as described by Arnold and Crandall.[1] A block of lead about 1.5 inches thick was placed against the face of the transmitter so as to form a cylindrical enclosure in front of the diaphragm, 1¾ inches in diameter and ⅜ inch long. The general arrangement is shown in Fig. 8. All crevices were sealed up so that the only openings to the cavity were two capillary tubes several inches long and of about 0.01 cm. bore. Two strips of gold foil were mounted symmetrically inside of this enclosure, the ends being clamped between small brass blocks. The supports were arranged in such a way that a current could be passed through the two strips in series. The connection between them was in electrical connection with the diaphragm.

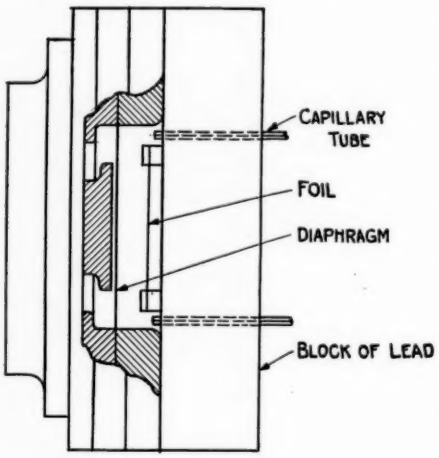


Fig. 8.

In the paper just cited it is shown that within an air-tight enclosure

$$\delta P = \frac{.0106 R i^2 P \sqrt{K_0} \left(\frac{\theta_1}{273}\right)^{1/4}}{\gamma V_0 \theta_2 f^{3/2}}, \qquad (12)$$

in which

δP = maximum value of the alternating pressure within the enclosure.
P = normal pressure within the enclosure.
R = resistance of the foil.
i = r.m.s. value of the alternating current passing through the foil.
K_0 = diffusivity at 0° C. of the gas within the enclosure.
θ_1 = mean absolute temperature of the foil.
θ_2 = mean absolute temperature of the gas.
γ = heat capacity per unit area of the foil.
V_0 = volume of the enclosure.
f = frequency of the alternating current.

Equation (12) may be used for calculating the pressure variation provided the wave-length of sound is large compared with the dimensions of the enclosure. The velocity of sound in hydrogen is about four times

[1] PHYSICAL REVIEW (Preceding Paper).

as great as in air; hence formula (12) holds for frequencies almost four times as high, when the enclosure is filled with hydrogen instead of air. Also, the diffusivity, K_0, is about six times as large for hydrogen as for air, so that greater pressure variation is obtained with the former. For these reasons, hydrogen was passed in a continuous stream through the enclosure by way of the capillary tubes, at a rate sufficiently slow to prevent any appreciable increase of the steady pressure above that of the atmosphere. The hydrogen was obtained from a Kipp generator and then passed through a solution of potassium permanganate and a drying tube containing phosphorus pentoxide.

In order to get the open circuit electromotive force of the transmitter, the circuit was arranged as in Fig. 9. The two resonant circuits were so

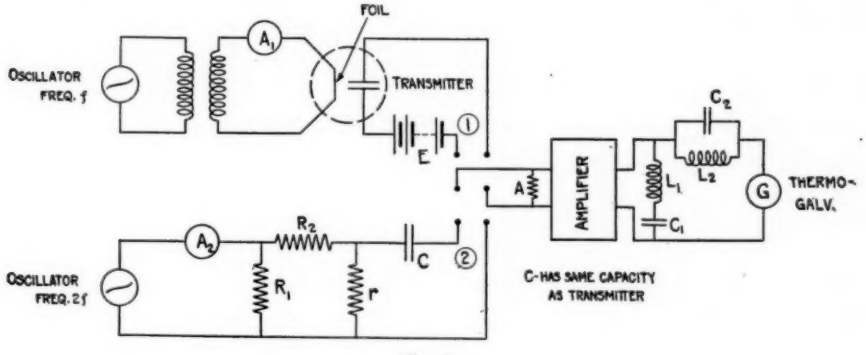


Fig. 9.

adjusted as to prevent current of the same frequency as that given by the oscillator from passing through the galvanometer. If a pure sine wave current passes through the foil, the pressure variation in the enclosure is of pure sine wave form and of double frequency. However, if there is any second harmonic present in the current, there will also be a component of the pressure variation of single frequency.[1] Putting in the resonant circuits eliminates this component from the measurements. The general procedure in making a measurement was as follows. The double-throw switch was first put in position 1, and the foil current and galvanometer current read. The switch was then thrown in position 2; R and r were then adjusted until the galvanometer read approximately the same as before. From the readings of A_2 and the values of R_1, R_2 and r, the voltage drop across r may be calculated. The open circuit voltage of the transmitter is then obtained by multiplying this voltage drop by the ratio of the galvanometer readings. That this gives us the open circuit voltage, follows from the fact that the transmitter

[1] Arnold & Crandall, *loc. cit.*

behaves as a generator having an internal impedance $1/C_0\omega$. Oscillator No. 2, of course, is set at double the frequency of Oscillator No. 1.

The current passed through the gold foil was about 0.5 ampere at all frequencies. Resistance measurements showed that with this current density, the foil was not heated more than 10° C. above the room temperature. The values of the quantities entering into the formula (12) for this experiment were as follows:

$R = 4.18$ ohms.
$P = 10^6$ dynes/cm^2.
$K_0 = 1.48$ C.G.S. units.
$\theta_2 = 295°$.
$\theta_1 = 305°$.
$\gamma = 4.15 \times 10^{-6}$ calories per sq. cm.
Thickness of gold foil $= 7 \times 10^{-6}$ cm.
Width of each strip = 1 cm.
Length of each strip = 2.6 cm.

Substituting these values in equation (12), we have for the root mean square value of the alternating pressure

$$2.59 \times 10^6 \frac{i^2}{f^{3/2}} \text{ dynes per sq. cm.}$$

Dividing the measured open circuit voltage by this value should give us the volts per unit pressure for all frequencies within certain limits. Measurements were made in this manner for frequencies from 160 to 18,000 cycles per second. The general shape of the curve obtained by plotting these values is shown in Fig. 10.

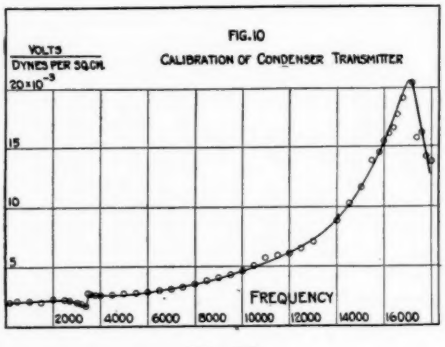


Fig. 10.

The absolute value of the sensitiveness at low frequencies as determined by this method was 0.121×10^{-3} volts per dyne, which is only about one sixteenth of that previously obtained by the piston method and by calculation from the dimensions of the instrument. In order to make further tests within the range of frequencies from 20 to 160 cycles, the gold was replaced by platinum foil, 4.42×10^{-4} cm. thick, and measurements were made as before. However, the size of the enclosure was increased in order to meet the conditions assumed in the derivation of formula (12), and for the same reason air was used instead of hydrogen.

Calculations made in a manner similar to that when gold foil was used gave a value of 1.93×10^{-3} volts per dyne per sq. cm. for the sensitiveness at low frequencies. This is in fair agreement with the value 2.00×10^{-3} obtained theoretically and with the piston apparatus.

Apparently when gold foil is immersed in hydrogen something takes place which is not taken account of in equation (12). The gold foil used was extremely thin (7×10^{-6} cm.) and when placed in hydrogen its specific heat per unit volume was apparently much greater than that of pure gold assumed in the calculations. On account of this discrepancy the gold leaf could not be relied upon for an absolute calibration, but it seemed reasonable to assume that the ratio between the true pressure and that calculated was independent of the frequency, so that a true relative calibration for different frequencies could be obtained. To get the absolute value of the efficiency at all frequencies, the values calculated from the readings on the gold foil were multiplied by the factor $2.0/.121 = 16.6$. The results so obtained are those plotted in Fig. 10.

6. Natural Frequency and Damping of the Diaphragm.

It is thought that this curve (Fig. 10) may be relied upon to give the sensitiveness in absolute value for frequencies up to 10,000 cycles. Above this frequency the wave-length of sound approaches the diameter of the cylindrical enclosure. The wave-length in hydrogen at 10,000 cycles is 13 cm. whereas the greatest distance from boundary to boundary of the enclosure is 4.4 cm. Although the absolute values of the sensitiveness above 10,000 cycles are probably not given by the points plotted in Fig. 10, nevertheless, this curve indicates in a general way the behavior of the transmitter at high frequencies. The principal peak in this curve comes at 17,000 cycles, which undoubtedly corresponds to the natural frequency of the diaphragm. The damping cannot be determined with any great assurance of accuracy, although the curve as drawn would indicate a damping factor of the vibrating system of about six or seven thousand.[1]

These high values of natural frequency and damping are in a large measure due to the cushion effect of the air between the plate and the diaphragm. Free lateral motion of the air is prevented by its viscosity. This increases the rate of dissipation of energy when the diaphragm is vibrating and also adds to its elasticity.

To see whether 17,000 cycles is a reasonable value for the natural frequency we may make an approximate theoretical calculation. When

[1] The term damping factor as here used may be defined as the reciprocal of the time required for the amplitude to fall to 1/2.718 of its initial value.

the frequency is as high as 17,000 cycles it seems reasonable to assume that there is practically no lateral motion of the film of air. Let us further assume that the film of air is compressed and rarified adiabatically by the motion of the diaphragm and also that the plate is of the same size as the diaphragm. This latter condition is not quite satisfied in the case of the electro-static transmitter but no great error is introduced by this assumption, since the motion near the edge of the diaphragm is small. Under these conditions if d is the length of the air gap, P, the atmospheric pressure, ρ, the mass per unit area of the diaphragm, and T, the tension, the equation of motion of the diaphragm becomes:

$$\rho \frac{d^2w}{dt^2} = T\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) - \frac{1.4Pw}{d},^1$$

or since w varies as $\epsilon^{j\omega t}$,

$$\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} + \left(\frac{\rho\omega}{T} - \frac{1.4P}{Td}\right)w = 0. \tag{13}$$

The solution of (13), consistent with the boundary conditions, is

$$w = J_0(lr),$$

in which

$$l = \sqrt{\frac{\rho\omega^2}{T} - \frac{1.4P}{Td}}.$$

The boundary conditions require that $J_0(la) = 0$. The lowest root of this equation is 2.4 so that

$$\left[\rho\omega^2 - \frac{1.4P}{d}\right]\frac{a^2}{T} = (2.4)^2$$

or

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{T}{\rho}}\left(\frac{2.4}{a}\right)\sqrt{1 + \frac{1.4Pa^2}{Td(2.4)^2}}.$$

This equation gives the natural frequency of the diaphragm when vibrating in its fundamental mode.

For the transmitter used in the preceding tests —

$$T = 6.57 \times 10^7 \text{ dynes per cm.}$$

$$\rho = .05 \text{ gm. per sq. cm.}$$

$$a = 2.18 \text{ cm.}$$

$$P = 10^6 \text{ dynes per sq. cm.}$$

$$d = 2.22 \times 10^{-3} \text{ cm.}$$

Hence

$$f_0 = 6{,}350\sqrt{8.9} = 19{,}000 \text{ P.P.S.}$$

[1] Rayleigh, Theory of Sound, L., 318.

which is slightly higher than the observed value. With the plate removed, the diaphragm would have a natural frequency of 6,350. This shows that the film of air between the plate and the diaphragm increases the elastic factor many times. It is due entirely to this fact that it has been possible to obtain natural frequencies above 10,000 without making the diaphragm exceptionally small.

We may satisfy ourselves that the maximum point in the efficiency curve is not due to resonance in the cylindrical enclosure by calculating its resonant frequencies. These frequencies are determined by the equation

$$J_n'(\sqrt{K^2 - p^2\pi^2 l^{-2}}\, R) = 0^1$$

in which

$$K = \frac{2}{a}\pi f_0.$$

a = velocity of sound.

l = length of cylinder.

R = radius of cylinder.

p = an integer.

Since the foil was placed symmetrically in the enclosure, only the symmetrical modes of vibration need be considered, in which case $n = 0$.

The first root of the equation $J_0'(Z) = 0$ is 3.83. For the lowest resonant frequency, $p = 0$, so that we have

$$f_0 = \frac{a}{2} \cdot \frac{3.83}{R}.$$

In this problem

a = 127,000 cm./sec. (velocity of sound in hydrogen).

R = 2.18 cm.

hence

f_0 = 35,500 cycles per second.

which is very much above the frequencies covered in the calibration.

If the enclosure is filled with air instead of hydrogen, the first resonant frequency comes at about one fourth of 35,500 or 9,000 p.p.s. A series of measurements were made with the circuit arranged as in Fig. 9, and air instead of hydrogen surrounding the gold foil. Points were calculated and plotted; the curve so obtained showed a sharp resonant point at 9,600 but none below. This may be taken as further evidence that the maximum point in Fig. 10 is not due to any resonance in the enclosure and so corresponds to the natural frequency of the diaphragm.

[1] Rayleigh, Theory of Sound, II, 300.

There is an irregularity in the calibration at about 3,500 periods per second. This is undoubtedly due to the natural frequency of the back-piece. At any rate, vibration of the plate would have an effect of this general character, *i. e.*, the efficiency would be decreased below, and increased above, resonance. In a later design, the plate and support have been made more rigid so as to form practically one solid piece. It is believed that with the newer model, the irregularity in the curve will have been eliminated.

This completes the account of the experimental work done in calibrating the instrument.

In order to obtain some idea of the sensitiveness of the electrostatic transmitter just described as compared with an electromagnetic instrument, the sensitiveness of the former was compared directly with an ordinary telephone receiver used as a transmitter, over a considerable range of frequencies. Except within a hundred cycles of the resonant frequency of the diaphragm of the receiver the electrostatic transmitter was found to generate a greater voltage for a given sound intensity.

7. Possibilities of Tuning.

Since an electrostatic transmitter is equivalent to an alternating current generator having an internal impedance $1/C_0\omega$, it is evident that, if in the circuit shown in Fig. 1, the resistance R is replaced by an inductance L, the voltage e will be a maximum for a frequency of

$$f = \frac{1}{2\pi\sqrt{LC_0}}.$$

The sharpness of tuning will of course depend upon the possibility of getting an inductance with a small resistance. In many problems in acoustics it is desirable to have a tuned system and in that case it is also better to have a diaphragm of low natural frequency and damping.

In order to get an expression for the sensitiveness as a function of the frequency, let us assume that we have a parallel plate condenser, one of the plates of which is fixed and the other moved perpendicularly to its own plane by a simple harmonic force. Practically this condition is approximated by a diaphragm, the center of which is separated a short distance from a plane plate as is shown in Fig. 2.

Let x = displacement of the diaphragm from its equilibrium position.

d = air gap, assumed large compared with x.

Then

$$\frac{1}{C} = \frac{1}{C_0}\left(1 + \frac{x}{d}\right).$$

The mechanical impedance of the diaphragm is

$$Z_1 = r + j\left(m\omega - \frac{s}{\omega}\right),$$

where r = resistance factor,

m = mass factor,

s = elasticity factor,

$\omega = 2\pi \times$ frequency.

If T = kinetic energy of the entire system,

W = potential energy of the entire system,

$2F$ = rate of dissipation of energy,

then

$$2T = m\dot{x}^2 + L\dot{y}^2,$$

$$2W = sx^2 + \frac{1}{C_0}\left(1 + \frac{x}{d}\right)(Y + y)^2,$$

$$2F = R\dot{y}^2 + r\dot{x}^2,$$

where Y is the permanent, and y the variable electric charge on the condenser. The equations of motion for the system are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial F}{\partial \dot{x}} + \frac{\partial W}{dx} = Pe^{j\omega t},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} + \frac{\partial F}{\partial \dot{y}} + \frac{\partial W}{\partial y} = 0.$$

If second order quantities are neglected, and also the constant terms, which affect only the equilibrium position, these equations become

$$\left.\begin{aligned} pm\dot{x} + r\dot{x} + \frac{s}{p}\dot{x} + \frac{Y\dot{y}}{pC_0 d} &= P \\ pL\dot{y} + R\dot{y} + \frac{1}{pC_0}\dot{y} + \frac{Y}{pC_0}\frac{\dot{x}}{d} &= 0 \end{aligned}\right\} \qquad (14)$$

in which p is written for

$$j\omega = \frac{d}{dt},$$

solving equations (14) for $\dot{y}$ and substituting the values,

$E = Y/C_0$, $Z_1 = (r + s/p + pm)$, $Z = (R + 1/pC_0 + RL)$,

we have

$$\dot{y} = \frac{PE}{pd[(E/pd)^2 - Z_1 Z]},$$

or

$$e = \dot{y}(R + pL) = \frac{PE(R + pL)}{pd[(E/pd)^2 - Z_1Z]}.$$

In any practical case $(E/pd)^2$ is small compared with Z_1Z so that we may write without much error

$$e = \frac{EP(R + pL)}{pdZ_1Z}.$$

In order to obtain a large value of e/P, which is a measure of sensitivness, Z_1Z should be made small, *i. e.*, the diaphragm should have a natural frequency equal to the frequency, $\omega/2\pi$, and the electrical circuit should be in resonance at the same point.

No extensive measurements have been made with the circuit arranged in this way, although enough has been done to show that it is feasible in some cases. The chief difficulty lies in the fact that the transmitter capacity is so small that the inductance has to be very large to get resonance for ordinary sound frequencies. This difficulty may be overcome by shunting the transmitter with a condenser, which of course reduces the generated voltage.

8. Characteristic Features of the Instrument.

Because of the high internal impedance of the electrostatic transmitter, it is possible to use the instrument efficiently only with high impedance apparatus, such as an electrostatic voltmeter or a vacuum tube amplifier. However, this is no special disadvantage if an amplifier is to be used, because it is not desirable to use a transformer in connection with an instrument for measuring sound intensities, since the ratio of transformation of a transformer is not independent of either frequency or load.

The method for calibration of this instrument as explained in the preceding pages is rather elaborate and requires considerable care. But since the efficiency depends primarily on the air gap and tension, it should not be difficult to make duplicate transmitters to which the same calibration applies, since the desired values of air gap and tension may be obtained without great difficulty, the former being tested by measuring the capacity, and the latter by determining the deflection produced by a known potential between the plate and diaphragm.

The fact that the sensitiveness of this instrument is independent of any properties of material, such as magnetization or electrical resistance, is of considerable advantage. For this not only allows us to make instruments which are almost exact duplicates, and so let the calibration for

one instrument serve for all the rest, but, the calibration is also constant with the time. The metal parts are of machine steel throughout; from the construction as shown in Fig. 2, it is therefore evident that temperature can affect the sensitiveness but little. The tension of the diaphragm is, of course, not absolutely independent of temperature, nor is the action of the cushion of air between the plate and the diaphragm independent of the barometric pressure: but these effects are hardly worth considering. Being made of heavy material, the transmitter satisfies the requirement in the way of ruggedness; having once been adjusted, it should remain so, even if subjected to considerable rough usage.

The sensitiveness of the transmitter is not absolutely uniform, but varies only about a hundred per cent. between zero and 10,000 cycles, as the curve in Fig. 10 shows. This variation is much less than would be the case with an electromagnetic instrument with a diaphragm having the same natural frequency and damping. Except for eddy current and iron losses, the voltage generated by an electromagnetic transmitter is proportional to the *velocity* of the diaphragm, whereas that given by the electrostatic transmitter is proportional to the *amplitude*. Below the natural frequency, the variation of velocity with frequency is much greater than the variation of amplitude since the velocity is proportional to the product of the frequency and amplitude.

In most problems the transmitter would be used with an amplifier. Now, the sensitiveness of the transmitter increases, whereas the efficiency of an amplifier sometimes decreases with the frequency; at any rate, it is possible to design a circuit for the amplifier, so that the combination of the two has a constant sensitiveness over a wide range of frequencies.

Since the natural frequency of the transmitter is very high, instantaneous records of sound waves obtained in combination with a distortionless oscillograph would not only give the relative amplitudes of the different frequencies into which the sound may be analyzed, but also the phase relations should be practically unchanged for frequencies up to 10,000 p.p.s.

As yet no instrument is available which will record without distortion currents of frequencies as high as 10,000 cycles. Only after such an instrument has been developed will it be possible to get a true record of consonant sounds. The same is true in regard to the quantitative study of the quality of musical instruments. However, by using an ordinary high frequency oscillograph in connection with a condenser transmitter and amplifier, it should be possible to get curves equal to or better than any obtained heretofore.

9. THE ELECTROSTATIC INSTRUMENT USED AS A STANDARD SOURCE OF SOUND.

There is of course no theoretical reason why the instrument described in the preceding pages cannot be used in a reversible manner: that is, as a source of sound when an alternating voltage is applied between the plate and the diaphragm. If the instrument is to be used in this way, it is better to have the plate the same size as the diaphragm, in order to get the maximum electrostatic force for a given voltage and air gap. The resulting increase in capacity is in general no disadvantage in this case. Also for convenience in using the instrument it may be desirable to have the face of the plate covered with a thin layer of mica.

Because of the simplicity of this type of instrument it is not difficult to calculate the output of sound energy for a given voltage *after its efficiency as a transmitter has been determined.* It is evident that the instrument can be excited in two different ways; (*a*) the alternating voltage can be applied alone, and (*b*) it may be superimposed on a static potential maintained by a battery in exactly the same way as when the instrument is used as a transmitter. The main principles underlying the two kinds of excitation in this case are quite similar to those discussed by Arnold and Crandall in connection with the excitation of the thermophone by pure A.C. and by A.C. with D.C. superimposed. For this reason neither type of excitation need be discussed at length; but a brief treatment of the condenser instrument excited by pure alternating current will be given.

When a pure alternating voltage is applied, the mean deflection of the diaphragm will depend on the magnitude of this voltage and the efficiency may vary somewhat because of the change in mean air gap, and the consequent change in the cushion effect of the air sheet on the motion of the diaphragm. It is therefore necessary to have curves corresponding to the curve in Fig. 10 but for a series of applied static potentials. These are most easily obtained by determining for a number of frequencies the generated voltage as a function of the static potential when sound of a fixed intensity falls on the transmitter. It will be found that the alternating voltage generated is so nearly proportional to the static potential that for most acoustic work this may be assumed to be the case.

When an alternating potential $\sqrt{2}v \sin \omega t$ is applied to the plates, the electrostatic force per unit area acting on the diaphragm is

$$\frac{v^2}{8\pi d^2}(1 - \cos 2\omega t). \qquad (15)$$

Now refer to Fig. 10, assuming that the curve there shown gives the

efficiency of the instrument, *used as a transmitter*, for an applied static potential v. If we multiply the ordinate (*i. e.*, the voltage per unit pressure) at frequency $\omega/\pi = f$ by the quantity

$$\frac{C_0}{v} \equiv \frac{C_0}{E}$$

we can obtain (cf. (4)) $\overline{C}_f$, the change in capacity per unit pressure. The total change in capacity due to the electrostatic force is then, (if $\overline{C}$ is the change per unit pressure at zero frequency)

$$C_1 = \frac{v^2}{8\pi d^2}(\overline{C} - \overline{C}_f \cos 2\omega t) \tag{16}$$

from which we can proceed to calculate the amplitude of motion of the diaphragm.

It is necessary of course to have a mean value of d, the air gap, but it is sufficiently accurate to take an arithmetic mean of the values at the center and at the edge of the diaphragm. The motion at the center is greater, but the motion near the edges extends over a greater area.

In computing the mean amplitude of the diaphragm we shall introduce very little error if we take the form of the diaphragm as that of a paraboloid. u, the amplitude at any radius, r, is given by the relation already quoted

$$u = \frac{P}{4T}(a^2 - r^2), \tag{17}$$

in which a = the radius of the diaphragm and plate. Equation (11) gives the total change in capacity in terms of P and T, that is (since $a = R$)

$$C_1 = \frac{a^4}{32d^2}\frac{P}{T}, \tag{11'}$$

or, eliminating P/T between (11') and (17) we have, for displacement at any radial distance r, in terms of maximum capacity change

$$u = \frac{8d^2}{a^4}(a^2 - r^2)C_1.$$

Substituting for C_1 the value given in (16) we have

$$u = \frac{v^2}{\pi a^4}(a^2 - r^2)(\overline{C} - \overline{C}_f \cos 2\omega t), \tag{18}$$

in which v is the r.m.s. value of the applied alternating voltage, and $\overline{C}_f$

is the change in capacity per unit pressure, determined in the manner described from the calibration curve of the instrument used as a transmitter. Equation (18) is rigorously true for all frequencies within the range of calibration, because the quantity $\overline{C}_f$ is taken from the calibration curve.

If, however, T is known, we can obtain an approximate value of u good at low frequencies, without any knowledge of C_1. (This is merely "equilibrium theory" and makes use only of the elastic factor, leaving the inertia and mechanical resistance of the moving system out of account). Substituting the value of electrostatic force (15) for P in (17) we have

$$u = \frac{v^2(a^2 - r^2)}{32\pi d^2 T}(1 - \cos 2\omega t). \tag{19}$$

The actual acoustic effect may be determined by the usual methods. If the diaphragm forms a wall of a small enclosure, the intensity is determined by the ratio of the volume displaced by the diaphragm as it vibrates to the volume of the enclosure. In other cases the intensity at a given point is calculated by determining the velocity potential due to the motion of the diaphragm.

It has been tacitly assumed that the amplitude of motion of the diaphragm is small compared with the air gap. This is necessary in order to get a pure tone when a sine wave E.M.F. is applied. While the instrument will not take care of a very large amount of energy, sound of the same order of intensity may be obtained as from an ordinary telephone receiver without appreciable distortion.

SUMMARY.

1. A description is given of a transmitter of the electrostatic type which is especially adapted for measurement of sound intensities over a wide range of frequencies. The instrument is portable and is sufficiently rugged to retain its calibration.

2. A discussion is given of the necessary auxiliary apparatus and the precautions necessary for proper use.

3. A theory of the transmitter has been developed by which its operation can be predicted from a few simple measurements.

4. A description is given of the calibration of such an instrument in absolute terms over a wide range of frequencies. It is found that its efficiency may be made practically uniform for frequencies up to 10,000 cycles per second, and the results of the calibration are in agreement with the theory.

5. The apparatus when once adjusted may be used for the measurement of the intensities of sound at any frequencies throughout this wide range without further special adjustment.

6. Due to the uniform response through this wide frequency range it will be possible to secure correct indications of complex wave forms and to determine not only the relative intensities of the components but also their phase differences.

7. When properly calibrated this apparatus can be used as a precision source of sound.

RESEARCH LABORATORY OF THE AMERICAN
TELEPHONE & TELEGRAPH CO. AND WESTERN ELECTRIC CO., INC.

THE RELATION OF OSMOTIC PRESSURE TO TEMPERATURE.

BY WILLIAM FRANCIS MAGIE.

II.

1. *Introduction.*—The present paper is an extension of one published under the same title in this REVIEW, Vol. XXXV., p. 272, 1912. In it the formula proposed in the former paper for the relation of the osmotic pressure to the temperature is deduced from a study of the boiling point and freezing point cycles, and certain improvements, consequent upon a more careful consideration of the quantities involved, are introduced. The formula is then tested by comparison with Emden's observations on vapor pressure.

The formula proposed in the former paper for the osmotic pressure was

$$p = a\theta \log \theta + (b - a)\theta + e \tag{1}$$

in which a is a quantity determined by observations of the heat capacity of the solutions, and b and e are constants of integration. On the assumption that a is independent of temperature, the e is connected with the heat of dilution l by the formula

$$l = -a\theta + e. \tag{2}$$

If we use p_s to designate the osmotic pressure at some definite temperature θ_s of the solution, the above formula becomes

$$\frac{p}{\theta} = \frac{p_s}{\theta_s} + a \log \frac{\theta}{\theta_s} - e \frac{\theta - \theta_s}{\theta\theta_s}. \tag{3}$$

It is in this form, with certain slight modifications, that the formula will hereafter be used.

2. *Osmotic Pressure.*—When the operation is considered by which a small mass Δm of solvent is forced out of a solution through a semipermeable membrane, thus reducing the volume of the solution by Δv, it is at once evident that the work done is not represented simply by $p\Delta v$. The solvent passes out against its own vapor pressure f, and if Δu represents the volume of the mass of solvent, negative work is done represented by $-f\Delta u$. Furthermore, work is done in compressing the solution by raising the pressure on it to p, which is in excess of the

negative work done when the pressure is removed, by an amount which is proportional to Δv, and may be represented by $h\Delta v$. The factor h is equal to $\beta p^2/2$, if we represent by β the compressibility of the solution. The work actually done in the osmotic pressure operation is therefore represented by

$$\left(p - f\frac{\Delta u}{\Delta v} + h\right)\Delta v = \Pi\Delta v. \tag{4}$$

The term containing h is generally insignificant, and we need to consider it only when extremely accurate measurements are available. In the examples which follow it amounts to only about 1/10th or 2/10ths of one per cent. of the whole. The vapor pressure term is practically independent of the concentration of the solution, and while it is insignificant at low temperatures and with strong solutions, it may become important at high temperatures with weak solutions.

Similarly the heat evolved and removed so that the osmotic pressure operation may be conducted isothermally contains a term which expresses the heat developed by compression. The total heat removed may be represented by

$$(\mu + \nu)\Delta v \tag{5}$$

in which μ represents the heat developed while the solvent is forced out, and ν the heat developed by compression.

3. *Boiling Point Cycle.*—A reversible cyclic operation may be performed by the aid of a semi-permeable membrane by (1) evaporating a mass Δm of solvent from a solution at the temperature θ_s; (2) lowering the temperature to θ_0, the boiling point of the solvent; (3) liquefying the vapor as solvent; (4) lowering the temperature to any temperature θ; (5) admitting the solvent into the solution through the semi-permeable membrane; (6) raising the temperature to θ_s. When the equations of energy and entropy are written out for this cycle, having regard to the fact that the volume of the mass Δm will be different at different temperatures, it proves to be impossible to obtain a relation between the osmotic pressures at the temperatures θ_s and θ which is symmetrical in respect to those temperatures, as it should be, so long as the ordinary and previously used definitions of a and l are accepted. If a and l are referred to change of mass instead of to change of volume this difficulty is removed. We shall adopt the definitions which yield the admissible form of the relation. Accordingly we define a as the rate at which the heat capacity of a system, consisting of solvent and of a solution containing one gram-molecule of the solute, changes as the mass of the solution increases by a transfer of mass from the solvent. Similarly we shall define l as the rate at which heat is evolved in a solution containing

one gram-molecule of the solute as its mass increases by the addition of more solvent. The experimental values of a and of l for aqueous solutions will not be materially changed by this change in definition.

When these definitions are used it becomes convenient to replace the term $\Pi\Delta v$, expressing the work done by the osmotic pressure, by the term $P\Delta m$, in which

$$P = \frac{\Pi}{\rho}\frac{\Delta v}{\Delta u}.$$

The symbol ρ represents the density of the solvent.

Similarly we can set the heat evolved in the osmotic pressure operation equal to $M\Delta m$, in which

$$M = \frac{\mu + \nu}{\rho}\frac{\Delta v}{\Delta u}.$$

By effecting the same change in a solution in two ways: (1) by the immediate introduction of a mass Δm of solvent, and the withdrawal of the heat $l\Delta m$ evolved; (2) by the admission into the solution of the same mass through a semi-permeable membrane, doing work $P\Delta m$ and supplying heat $M\Delta m$, it is easy to show that

$$M = P - l. \tag{6}$$

With these definitions, and on the assumption that a is independent of the temperature, so that it may be considered constant in the integrals that appear in the entropy equation, we obtain from the combination of the energy and entropy equations, with the use of Equations (2) and (6), the formulas

$$\frac{P}{\theta} = \frac{P_s}{\theta_s} + a\log\frac{\theta}{\theta_s} - e\frac{\theta - \theta_s}{\theta\theta_s}. \tag{7}$$

$$\frac{P_s}{\theta_s} = \lambda_0\frac{\Delta\theta}{\theta_0\theta_s} - \frac{s - \sigma}{2}\frac{\Delta\theta^2}{\theta_s^2} - f\left(\frac{1}{\rho_0} - \frac{1}{\rho_s}\frac{\Delta v_s}{\Delta u_s}\right). \tag{8}$$

In Equation (8), we have denoted by λ_0 the latent heat of the solvent at the temperature θ_0, by s and σ the specific heats of the liquid solvent and of its vapor, by $\Delta\theta$ the elevation of the boiling point of the solution, or $\theta_s - \theta_0$; and by f the pressure of the vapor. The other symbols have already been defined. The last term in (8) is insignificant in practice.

4. *Freezing Point Cycle.*—By the use of a similar cycle carried through the freezing points of solution and solvent, Equation 7 can be obtained, and Equation 8 also, with the omission of the insignificant last term. In Equation 8 the symbols represent those quantities, analogous to those defined in connection with that equation, which are appropriate

to the process of melting. A cycle of this kind has been studied by Ewan.[1] The approximations which Ewan employs are such that his formula will give results slightly different from those given by Equation 7.

5. *Test by Emden's Observations of Vapor Pressure.*—These formulas can be tested by applying them to Emden's observations on vapor pressure.[2] Emden observed the vapor pressure over pure water and over various aqueous solutions within a range of temperature from about 12° C. to 95° C. He found that the pressures over water could be well represented by the formula proposed by Magnus

$$f = f_0 10^{bt/c+t},$$

and that the pressures over the solutions were represented by a similar formula, in which the constants b and c were unchanged, and the constant f_0 took a new value f_0'. Thus, according to Emden, the ratio

$$\frac{f'}{f} = \frac{f_0'}{f_0}$$

at all temperatures, and von Babo's law holds for these pressures. Since our formula for osmotic pressure shows that von Babo's law cannot be accurately correct for all temperatures, we must calculate the vapor pressures to which our formula will lead, in order to see whether or not they also agree with Emden's observations within the limits of his experimental errors. To do this (1) we first calculate from Emden's formula the temperature t' at which the solution has the same vapor pressure as water at certain definite temperatures t, and thus obtain a set of values of $t' - t = \Delta\theta$, the elevations of the boiling point at these temperatures. (2) We then select one of these values of $\Delta\theta$ as correct, preferably taking one for which the ratio f'/f given by the experimental numbers is the nearest to the ratio f_0'/f_0 used by Emden in calculating the pressures over the solution for comparison with the observed pressures. The particular $\Delta\theta$ taken was in each of the cases following that for which the temperature of the water was 80° C. (3) With this value of $\Delta\theta$ we calculate P_s/θ_s for that temperature. In doing this the term containing the vapor pressures may be neglected. The values of λ_0 employed in the examples following were calculated with a formula kindly furnished me by Professor Harvey N. Davis, of Harvard University, according to which the latent heat of water vapor at any temperature t° C. is given by

$$\lambda = 92.98[374.5 - t]^{0.3133}.$$

(4) From this value of P_s/θ_s and the values of a and of l obtained for the

[1] Ewan, Zeitsch. f. Phys. Chem., XXXI., p. 22, 1899.

[2] Emden, Wied. Ann., XXXI., p. 145, 1887.

appropriate concentrations from observations of heat capacities[1] and heats of dilution[2] the values of P/θ are calculated for the other temperatures at which comparisons are to be made. The values of θ_s obtained from Emden's formulas are sufficiently accurate to be used in the small terms of the formula for P/θ. If they are not known, that is, if we have but one well determined value of $\Delta\theta$ from which to calculate P_s/θ_s, we can calculate P/θ by successive approximations. (5) Having the values of P/θ for different temperatures we now calculate back by the use of the formula for P_s/θ_s to the values of $\Delta\theta$ for these temperatures. These values generally differ only a little from those obtained from Emden's formulas. Since for small temperature ranges the curve plotted with the values thus obtained for $\Delta\theta$, measured off along the temperature axis from the vapor pressure curve for water vapor, and the curve similarly plotted with Emden's values of $\Delta\theta$, will be closely parallel, the true differences between the vapor pressure over the water and the vapor pressure over the solution at the same temperature will be proportional to that obtained from Emden's formula in the ratio of the two values of $\Delta\theta$. Thus we have

$$\frac{\Delta\theta}{\Delta\theta_e} = \frac{f - f'}{f - f_e'} = \frac{1 - f'/f}{1 - (f_0'/f_0)_e}$$

using the subscript e to designate Emden's numbers. We may thus calculate the true ratio f'/f and from this, f'. The vapor pressure thus calculated for the different temperatures may then be compared with the experimental values, or with those calculated from Emden's formula, which agree with the experimental values within the limits of the experimental error.

The following tables exhibit the results of such calculations. The solutions for which we have at present the necessary information to enable us to carry out the calculations are those for (I.) sodium chloride 5.067 parts, (II.) sodium chloride 10.096 parts, and (III.) potassium chloride 10.051 parts, dissolved in 100 parts of water. The temperatures of the boiling points of the pure water are given in Centigrade degrees. In making the calculations, 273.1 was taken as the temperature of the Centigrade zero on the absolute scale. The unit of energy is the calorie and therefore the osmotic pressure is measured by a number which can be reduced to absolute units by multiplication by J.

A glance at the figures in the last two columns of these tables will show that the formula presented for the osmotic pressure furnishes values of the vapor pressure over the solutions which agree very closely with those calculated from Emden's formula.

[1] Magie, this REVIEW, XXV., p. 171, 1907.

[2] Magie, this REVIEW, XXXV., p. 272, 1912.

1. *$\Delta\theta$ from Emden's formula.*

Temp. of Water.	Emden's f for Water.	I. $\Delta\theta$.	II. $\Delta\theta$.	III. $\Delta\theta$.
0	4.5625	0.4549	0.9179	0.6100
20	17.461	0.5351	1.0800	0.7176
40	55.035	0.6219	1.2554	0.8340
60	148.581	0.7152	1.4439	0.9592
80	353.894	0.8151	1.6459	1.0932
100	760.	0.9215	1.8609	1.2360

2. *Values of a and e.*

	a	*e*
I.	− 0.005671	− 1.806
II:	− 0.01662	− 5.405
III:	− 0.01376	− 4.3084

3. *P/θ, $\Delta\theta$ calculated from formula for $\Delta\theta$, f' calculated from this $\Delta\theta$, f_e' calculated from Emden's formula. The numbers in brackets are those on which the calculation was based.*

I.

Temp. of Water.	P/θ.	$\Delta\theta$.	f'.	f'_e.
	0.00			
0	35609	0.4470	4.418	4.414
20	36105	0.5313	16.897	16.893
40	36292	0.6207	53.250	53.246
60	36235	0.7152	143.75	143.75
80	[35992]	[0.8151]	342.39	342.39
100	35601	0.9204	735.33	735.30

II.

	0.00			
0	70374	0.8852	4.279	4.269
20	72093	1.0631	16.354	16.337
40	72861	1.2490	51.511	51.492
60	72891	1.4424	139.03	139.02
80	[72474]	[1.6459]	331.11	331.11
100	71607	1.8552	711.2	711.1

III.

	0.00			
0	47929	0.6021	4.368	4.365
20	48949	0.7208	16.701	16.704
40	49230	0.8426	52.63	52.65
60	48947	0.9670	142.10	142.14
80	[48227]	[1.0932]	338.56	338.56
100	47163	1.2205	727.5	727.1

6. *Comparison with Osmotic Pressures Obtained by other Methods.*— The true osmotic pressures p can be calculated from the formulas defining

P, given in § 1. It did not seem necessary to present them. They do not differ from the corresponding values of ρP by more than two per cent. in the extreme case of the highest temperature. They do not show an exact proportionality with θ, as we should expect them to do, if the osmotic pressure in all cases obeys the laws of gases. Numerically they are so great, amounting, for example, to 40.28 atmospheres at 0° C., for the weaker sodium chloride solution, as to indicate the presence of more than two molecules in the volume of the water used. This result is inconsistent with the ordinary form of the dissociation hypothesis, and points to some form of the association theory as necessary for its explanation. Similar results have been found by Kahlenberg[1] in some cases by the study of freezing and boiling points.

Furthermore, the ratios of the true osmotic pressures of the two sodium chloride solutions, instead of being equal to 1.9925, as they should be if the osmotic pressure is proportional to the mass of dissolved solute, or even less than that, if the more dilute solution is more dissociated, and the osmotic pressure is proportional to the extent of dissociation, are in all cases except at 0° C. slightly greater than 2, being generally equal about to 2.010.

Kahlenberg's observations on the freezing points of sodium chloride solutions may be used for comparison with the results calculated from vapor pressure. The freezing points observed, plotted against the quantities of salt dissolved, determine a straight line, from which we find that the depression of the freezing point for our solution I. of sodium chloride should be 2.990°. From this we can calculate the value of P_s/θ_s at that temperature. The value of P/θ obtained for 0° C. from this freezing point is 0.0032086, and is about 10 per cent. lower than that given in Table 3., I., for approximately the same temperature. If it were the correct value it would require a vapor pressure over the solution at 0° C. of 4.431. The difference between this pressure and the pressure 4.418 calculated by the formula for vapor pressure is within the limits of experimental error; yet it cannot be said that Kahlenberg's observation confirms our formula. Dieterici's[2] direct observation of the vapor pressure over sodium chloride solutions at 0° C. also gives 4.432 for a solution of the strength we are considering. The value of the osmotic pressure calculated from Kahlenberg's freezing point is in fair agreement with the dissociation hypothesis in this instance. Kahlenberg's boiling points give also lower values in the case of sodium chloride solutions than those calculated from Emden's observations by

[1] Kahlenberg, Jour. of Phys. Chem., Vol. V., p. 339, 1901.

[2] Dieterici, Wied. Ann., LXVII., p. 859, 1899.

our formula. From Kahlenberg's series 3 we find the elevation of the boiling points under 754 mm. pressure to be 0.820 for solution I., 1.720 for solution II. The boiling point elevations determined from the formula of this paper for the same pressure are for solution I., 0.9194, for solution II., 1.853. They are inconsistent with the dissociation hypothesis.

In the case of potassium chloride solutions Kahlenberg's Series 3 gives for the elevation of the boiling point of a solution like solution III., 1.27. The value calculated from our formula is 1.214.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE WASHINGTON MEETING.

THE eighty-ninth meeting of the Physical Society was held at the National Bureau of Standards, Washington, D. C., April 20 and 21. Two sessions for the reading of papers were held on Friday and one on Saturday.

The following program was presented:

Thermal Expansion of Marble. L. W. SCHAD.

The Composition of Speech. I. B. CRANDALL.

Polarization at the Cathode in Oxygen. C. A. SKINNER.

The Energy of Emission of Photo-electrons from Film-coated and Non-homogeneous Surfaces: A Theoretical Study. A. E. HENNINGS.

The Electrical Conductivity of Sputtered Films. R. W. KING.

Elastic Impact of Electrons with Helium Atoms. J. M. BENADE.

The Loss of Energy of Wehnelt Cathodes by Electron Emission. W. WILSON.

Theory of Ionization by Partially Elastic Collisions. K. T. COMPTON.

The Passage of Low-Speed Electrons through Mercury Vapor and the Ionizing Potential of Mercury Vapor. JOHN T. TATE.

Amplification of the Photoelectric Current by the Audion. JACOB KUNZ.

The Reflection Coefficient of Monochromatic X-Rays from Rock Salt and Calcite. A. H. COMPTON.

The Measurement of "h" by Means of X-Rays. F. C. BLAKE AND WILLIAM DUANE.

The Crystal Structure of Magnesium. A. W. HULL.

On the Ionization Potentials of Vapors and Gases. J. C. MCLENNAN.

The Necessary Physical Assumptions Underlying a Proof of Planck's Radiation Law. F. RUSSELL V. BICHOWSKY.

High Vacuum Spectra from the Impact of Cathode Rays. LOUIS THOMPSON.

A New Theory concerning the Mathematical Structure of Band Series. R. T. BIRGE.

The Wave-length of Light from the Spark which Excites Fluorescence in Nitrogen. CHARLES F. MEYER.

A Measuring Engine for Reading Wave-lengths from Prismatic Spectrograms. L. G. HOXTON.

Natural and Magnetic Rotation at High Temperatures. FREDERICK BATES AND F. P. PHELPS.

The Significance of Certain New Phenomena Recently Observed in Preliminary Experiments on the Temperature Coefficient of Contact Potential. A. E. HENNINGS.

Generalized Coördinates, Relativity and Gravitation. E. B. WILSON.

The Motion of an Aëroplane in Gusts. E. B. WILSON.

Optical Constants by Reflection Measurements. L. B. TUCKERMANN, JR., AND A. Q. TOOL.

The High-Frequency Absorption Bands of Some of the Elements. F. C. BLAKE AND WILLIAM DUANE.

A Proposed Method for the Photometry of Lights of Different Colors. III. IRWIN G. PRIEST.

The Use of a Thomson Galvanometer with a Photoelectric Cell. W. W. COBLENTZ.

On the Occurrence of Harmonics in the Infra-Red Absorption Spectra of Gases. W. W. COBLENTZ.

On Friday afternoon President Millikan outlined to the Society the plans of the National Research Council for enabling the United States government to utilize the research ability of this and other scientific organizations for the national defence. Considerable discussion followed.

At a short business session on Friday morning the following amendment to the By-laws, presented from the Council, was adopted without opposing vote. Change By-law No. 1, section 2, to read as follows:

"For the election of a new member to the Society, either regular, honorary or associate, or for the transfer of an associate member to regular membership, it shall be necessary that a proposition in due form signed by two members of the Society shall be presented at a meeting of the Council, and that at a meeting of the Council occurring at least two weeks later, the person named in such proposition shall receive the favorable ballots of a majority of the members present. The council may, however, at its discretion make election to associate membership at the same meeting at which the nomination is presented."

The secretary made a brief report on the Pacific Coast meeting held at Leland Stanford, Jr., University on April 8 under the direction of Pacific Coast Secretary Lewis.

The attendance at the three sessions of the Washington meeting was about 110, 140 and 100 respectively. Visiting members were the guests of the Washington members for lunch on both days. A subscription dinner at the Cosmos Club on Friday evening was enjoyed by about fifty.

A cordial vote of thanks was extended to the Washington members for the excellent arrangements for the meeting and the various courtesies extended.

The October meeting of the Society will be held at Rochester, N. Y., and an excellent opportunity will be given to become acquainted with the important industrial research laboratories there.

A. D. COLE,
Secretary.

THERMAL EXPANSION OF MARBLE.[1]

BY LLOYD W. SCHAD.

A RECENT investigation on various American marbles shows that the coefficient of thermal expansion of marble increases from about 1×10^{-6} at 0° C. to 28×10^{-6} at 300° C. After a marble has been expanded by heat it does not come back to its original dimensions but a permanent increase results, the magnitude of which depends upon the temperature to which the specimen has been heated. A permanent increase in length amounting to as much as 0.4 per cent. has resulted from heating a specimen to 300° C.

NATIONAL BUREAU OF STANDARDS,
WASHINGTON, D. C.

THE COMPOSITION OF SPEECH.[1]

BY I. B. CRANDALL.

THIS paper deals with the composition of average speech from sounds of different frequencies, speech being considered as a continuous flow of distributed energy, analogous to total radiation from an optical source. This idea of speech is a convenient approximation, useful in the study of speech reproduction by mechanical means.

Two properties characterize perfect speech reproduction: (1) The accurate transfer of the language used, and (2) the preservation of the tone-quality of the original speech. For expressing the ideal property of literal accuracy in transfer various terms have been used, such as, "clearness," "intelligibility," "articulation," and so on. The term "articulation" will be chosen to describe this property of literal reproduction.

In reference to the other idea, namely, the preservation of the tone-quality of speech, the term "naturalness" will be used. The idea of naturalness includes the preservation of the human or artistic quality of speech.

Consider first the relative importance of the different speech frequencies from the standpoint of articulation. Before we can determine this factor, we must have a method of measuring articulation; this involves the choice of a number of representative sounds, and the adoption of a testing routine which will give the per cent. of such sounds accurately transferred by the reproducing apparatus.[2] For a first attempt a list of the representative consonant sounds in the English language has been taken, for experience has shown that it is possible to identify most words in a given context without taking notice of the vowels. The routine of articulation testing is rather tedious and need not be gone into, as we are only interested in the result—

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] The method used is a development of that originally proposed by Dr. G. A. Campbell in his article on "Telephonic Intelligibility," Phil. Mag., 19, 1910, p. 152.

the per cent. of consonant sounds accurately transferred by any given apparatus.

It is possible to measure the relative importance of different speech frequencies if we make a series of articulation tests using apparatus which completely suppresses certain frequencies, while at the same time the remaining frequencies are perfectly reproduced. It suffices to state here that electromechanical reproducing systems are available which have exactly these characteristics.

Denoting the *importance to articulation* of any frequency p by the function $D(p)$, we may consider the articulation χ of a system which reproduces all frequencies equally to be

$$\chi_{\max.} = \int_0^\infty D(p)dp = 1.00.$$

By measuring χ when different, limited ranges of frequencies are reproduced, it is possible to find $D(p)$. This method has been worked out in detail by the writer and has yielded a good determination of the relative importance $D(p)$ of the different frequencies which compose the consonant sounds.

Coming now to the question of naturalness, the tone-quality of speech is clearly defined if the relative amounts of energy associated with the different frequencies are known: for this purpose we make use of another function of frequency $S(p)$ which indicates the *energy distribution* in speech. The composition of one unit of speech energy from energy of different frequencies may be expressed by

$$\int_0^\infty S(p)dp = 1.00.$$

Measurements of the relative intensities of different sounds are readily carried out, and it is possible to determine the function $S(p)$ in a number of ways. One way would be to use apparatus similar to that used for the determination of $D(p)$ in which certain frequencies were absolutely suppressed. Instead of measuring articulation, we should measure the loss in loudness or energy corresponding to a given suppression, from which data $S(p)$ could be easily found. Another way would be to experiment with systems which reproduce all frequencies, but which overemphasize certain ranges of frequency. Some rough experiments of this kind have been made from which preliminary values of the function $S(p)$ have been obtained.

The interesting thing, in the energy distribution in speech, is that the vowels are the determining factors of this distribution, whereas the consonants are the determining factors in the matter of importance to articulation. The importance of the consonant frequencies in speech is thus utterly out of proportion to the amount of energy associated with them.

On account of the fact that the energy in speech resides almost wholly in the vowel sounds, it is possible to obtain the curve $S(p)$ synthetically if the energy distributions in the different vowel sounds are known. Making use of

Professor Miller's well-known results for energy distribution in the vowel sounds and weighting each vowel for frequency of occurrence, I have constructed such a synthetic curve; the agreement between the synthetic curve and the experimental values obtained from speech as a whole is practically complete. More accurate data for the energy distribution will be offered in a subsequent paper.

Because of the small amount of energy in the consonant sounds, they are difficult to investigate; but experiments are in progress from which we hope to obtain an interpretation of what has been called the *importance* of a given consonant frequency. It is also hoped to give a complete treatment of the reproduction of speech, based on the idea of the composition of speech given in the present paper.

RESEARCH LABORATORY OF THE AMERICAN
TELEPHONE AND TELEGRAPH COMPANY
AND WESTERN ELECTRIC COMPANY, INC.

POLARIZATION AT THE CATHODE IN OXYGEN.[1]

BY C. A. SKINNER.

IN this paper were reported measurements of the polarization at the cathode in oxygen, similar to those already published for hydrogen.[2]

These give the "transverse" current between two small electrodes placed in the negative glow of a separate "ionizing" current, for given differences of potential maintained between these "transverse" electrodes. The published articles show that this applied difference of potential is practically concentrated between the transverse cathode and the gas.

The results from oxygen are similar to those from hydrogen. Expressed in terms of the apparent resistance at the transverse cathode, they are briefly summarized as follows:

For a given ionizing current, the apparent cathode resistance remains practically constant as the polarizing P. D. is increased from 0 to about 15 volts. Between about 15 volts and 140 volts the resistance is proportional to the transverse current, giving the relation

$$V_b \propto j^2,$$

where V_b is the polarization P. D. and j the transverse current density. Above 140 volts polarization P. D. the resistance rises to a maximum value then slowly decreases—supposedly because of the increased production of ions in the negative glow by the electrons escaping from the transverse cathode.

The deviation of the resistance curve from a straight line for polarization potentials below 15 volts is ascribed to the temperature velocity of the ions discharging to the cathode.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] PHYS. REV., February and April, 1917.

For a given polarization P. D. the apparent cathode resistance is practically inversely proportional to the ionizing current density. From this law, the calculated cathode resistance per cm.^{2} at the transverse cathode is found to be the same as that determined at the main cathode by wire sound measurements.

With transverse cathodes of fine wires, their apparent resistance per cm.^{2} is found to be, for the same polarization P. D., smaller than for plane cathodes. Comparing different sizes of wires, the cathode resistance is approximately inversely proportional to the square root of their radius—if the same polarizing P. D. be used.

University of Nebraska,
April, 1917.

Elastic Impact of Electrons with Helium Atoms.[1]

By J. M. Benade.

It is well known that in ordinary gases an electron colliding with a molecule loses all or nearly all its kinetic energy, even though its velocity is less than the critical amount necessary for ionization. It has been generally assumed and not without reason that in monatomic gases the impacts are perfectly elastic. This means that when a collision occurs without ionization, no vibration is set up within the atom and that no rotary motion is imparted to the atom.

It is to be expected that the impinging electron will impart to the atom a small velocity, thereby losing kinetic energy even in the case of perfectly elastic impact. The influence of thermal motions of the atoms is negligible. The average loss of energy assuming perfect elasticity of impact is 0.0054 volts when the velocity of the electron corresponds to a 20-volt drop, whereas Frank and Hertz estimated an average loss of 0.3 volt per collision in helium.

An attempt has been made to determine the loss of energy per collision in very pure helium with the result that the observed fraction of the electron's kinetic energy which is lost is in close agreement with value calculated assuming perfect elasticity. The agreement is such as to indicate that there is no energy lost by inelasticity of impact.

The method employed was to determine the minimum voltage required to cause ionization by collision in helium between two parallel electrodes at one of which electrons were liberated by ultra-violet light. When the distance between the electrodes, and the gas pressure, are small the necessary voltage is the well-known minimum ionizing potential of helium (20 volts). But as the distance and pressure are increased, the voltage, at which ionization begins, rises indefinitely. At a pressure of 43.4 mm. and distance between electrodes of 12.70 mm. the voltage necessary to cause ionization is 110.

The fraction of energy lost by an electron at a collision is $(1 - B)$ in the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

formula for the sum of a geometrical series

$$S = A\left(\frac{1 - B^n}{1 - B}\right),$$

in which S is the energy of an average electron just able to ionize after making n collisions, gaining A volts between successive impacts. n is obtained from an extended table given by K. T. Compton's theory of ionization by collision in the case of elastic impact. A is obtained by dividing the total voltage by n. Since $B = .99973$, B^n can not be neglected unless n is very large, when $(1 - B)$ becomes equal to A/S. In this case the gain between collisions is equal to the loss at collision.

It has also been found that for helium X/p_m (Stoletow's constant) is not a constant quantity as in gases where collisions are inelastic. Values of X/p_m for helium range from 10 to 27.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
April 7, 1917.

THE ENERGY OF EMISSION OF PHOTO-ELECTRONS FROM FILM-COATED AND NON-HOMOGENEOUS SURFACES: A THEORETICAL STUDY.[1]

BY A. E. HENNINGS.

THIS study extends the theoretical considerations and examines analytically certain of the suggestions contained in a recent paper.[2] It deals with the contact P. D. of a non-homogeneous surface brought into existence by the formation of a photo-electrically active film upon a similarly active homogeneous surface and the corresponding maximum energies of emission of electrons from the components of such a composite surface when stimulated by light of a given frequency. If the velocity of escape of electrons from one surface or surface element to another is affected by the presence, in the near proximity of one of them, of surfaces or surface elements of a different character, the logical conclusion seems to be that the potential to be applied in order just to prevent the escape of electrons from one of the component portions of a composite surface may not be at all the same as that required to prevent their escape from a surface made up entirely of this component. An active, initially homogeneous, surface upon which an active film electronegative to it is forming might discharge electrons with abnormally high velocities, while the electronegative film, at least in the earlier stages of its formation, might discharge electrons with abnormally low velocities. Certain characteristic features of the "distribution of velocity" curves for a series of metals or for a given metal under different surface conditions lend support to the point of view which forms the basis of the analytical argument.

UNIVERSITY OF SASKATCHEWAN,
April, 1917.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] Hennings and Kadesch, PHYS. REV., N.S., 8, p. 209, 1916.

The Loss of Energy of Wehnelt Cathodes by Electron Emission.[1]

By W. Wilson.

It was first shown by O. W. Richardson in 1903 that the thermionic current from a hot cathode is given by the equation

$$i = a\theta^{1/2}e^{-b/\theta},$$

where θ is the absolute temperature and a and b are constants.

The constant b has the special significance that it is proportional to the work done by an electron in leaving the surface of the body in question. This work can be determined directly by measuring the difference in power required to maintain a body at a certain temperature when it is emitting electrons from when it is not. Richardson and Cooke and later Lester have obtained values for tungsten which are in very good agreement with the values of b obtained by Langmuir and K. K. Smith.

On the other hand, Wehnelt & Jentsch, Schneider, Wehnelt and Liebrich, and Richarsdon & Cooke have all found that for lime-covered cathodes either the effect is so small in comparison with other energy changes as to be completely masked or that there is no correspondence between the two quantities.

Since these experiments suggest that the mechanism of thermionic emission from Wehnelt cathodes is different from that for pure metals further experiments were made by the author to determine whether consistent results could be obtained by using more stable cathodes.

If W is the work done by an electron in leaving the surface of a hot body $W = bR$, where R is the gas constant for one molecule.

If it is assumed that the work done by the electron is that done by moving through a double layer of strength ϕ we have $W = \phi e$ and $\phi = bR/e$.

The method of Richardson and Cooke was used for the direct determination of ϕ. The constant b was determined in the usual manner, the thermionic current being measured with the cathode at different temperatures which were obtained by means of an optical pyrometer of the Holborn and Kurlbaum type.

The following are the results obtained:

Fil. No.	b Degrees.	bR/e Volts.	θ Volts.
1	27,200	2.34	2.39
2	30,100	2.59	2.54
3	23,500	2.02	1.97
4	25,200	2.16	2.28
5	38,200	3.28	3.22

Filaments 1 and 2 were coated with BaO 50 per cent., SrO 25 per cent., CaO 25 per cent., Filaments 3 and 4 with BaO 50 per cent., SrO 50 per cent., and Filament 5 with CaO alone.

It appears that for Wehnelt cathodes the values ϕ and bR/e show a good

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

correspondence. This is a strong point in favor of the view that the emission of electrons from Wehnelt cathodes is due to a similar mechanism to that causing the emission from heated pure metals. It is also a further proof of the substantial correctness of Richardson's hypotheses to account for the emission of electrons by hot bodies.

RESEARCH LABORATORY,
WESTERN ELECTRIC CO., NEW YORK.

THEORY OF IONIZATION BY PARTIALLY ELASTIC COLLISIONS.[1]

BY K. T. COMPTON.

IN a recent paper[2] the writer showed that, in a gas in which the electrons lose no energy at impacts with molecules, except when these impacts result in ionization, there should be a functional relation

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p}\right)$$

between the average number of ionizations α per electron per unit path, the pressure p and the electric field intensity X. The equation expressing this relation was derived from a simple assumption regarding the probability of ionization at a collision.

Townsend has shown that, if there is any functional relation between α/p and X/p, there should be a definite value for the Stoletow constant X/p_m for the gas, where p_m refers to the gas pressure at which the current through the gas is maximum. Mr. Benade has shown that no such constant exists in the case of helium, and also that the collisions in helium result in a small loss of energy. His results further indicate that this loss of energy is entirely accounted for by the velocity imparted to the molecules by the colliding electrons, and therefore that the collisions are really perfectly elastic in their nature.

The equations developed in this paper cover this case and also any case in which the collisions are slightly inelastic. If ν represents the total number of collisions made by an electron while advancing unit distance in the direction of X, and if c is the average energy, in equivalent volts, lost at a collision, then $X' = X - c\nu$ represents the net gain of energy per unit path. Thus this case may be treated as a case of impact in which no energy is lost if we put

$$\frac{\alpha}{p} = \psi\left(\frac{X'}{p}\right) = \psi\left(\frac{X - c\nu}{p}\right).$$

If $\nu_1 = pN$ is the number of collisions per unit path in any direction, $\nu_1^2/\nu^2 = (P + \delta)$, where δ is the ratio of c to the minimum ionizing potential V_0. We can therefore put

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p} - \frac{2cN^2}{\frac{\alpha}{p} + \sqrt{\frac{\alpha^2}{p^2} + 4N\delta}}\right).$$

This equation is accurately verified by the experimental results of Gill and Pidduck and it is found that $c = 0.00268$ volt, $V_0 = 20.99$ volts, $N = 8.7$ collisions per centimeter path at a pressure of one millimeter.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY.

THE PASSAGE OF LOW SPEED ELECTRONS THROUGH MERCURY VAPOR AND THE IONIZING POTENTIAL OF MERCURY VAPOR.[1]

BY JOHN T. TATE.

THE present investigation is an attempt to determine with greater precision the value of the ionizing potential of mercury vapor, and, in general, to arrive at some explanation for the phenomena observed in connection with the passage of low-speed electrons through mercury vapor. Franck and Hertz have shown that the impacts between electrons and mercury molecules become inelastic when the velocity of the electrons reaches a value corresponding to a fall in potential of 4.9 volts. They interpreted their results as indicating ionization of the mercury vapor at that potential but, as the writer has pointed out, their results indicate nothing but inelasticity of impact. On the other hand the results of Newman and of Gaucher would seem to indicate the beginning of a weak ionization at that point. Some time ago, however, the writer was able to show that if there were ionization at 4.9 volts it was of relatively infinitesimal magnitude as compared with the ionization which takes place at about 10 volts, and further showed that this was the minimum potential required for the production of the many-lined spectrum of mercury.

The apparatus used in this investigation was similar to that used by Franck and Hertz. The source of electrons was a lime-covered platinum wire which formed the axis of two coaxial cylinders, the inner one of platinum net and the outer of platinum foil. Varying accelerating potentials were applied between the hot wire and net cylinder and measurements made simultaneously of the total current between the hot wire and cylinders and of the current reaching the outer cylinder against a retarding field. Measurements of this type were carried out under varying conditions of vapor pressure of mercury and of temperature of wire.

In general, the *potential-total current* curves show the characteristic discontinuity which takes place at a potential (usually greater than 10 volts) depending upon the temperature of the wire and upon the vapor pressure. The higher the temperature of the wire the lower the potential at which the break occurs. Simultaneously with the sudden increase in current a visible

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

glow appears in the neighborhood of the hot wire and there is a copious formation of positive ions as indicated by the large positive currents between the cylinders. With very hot wires the discontinuity could be made to take place at potentials as low as 7.5 volts. After the ionization has set in it is possible to decrease the potential considerably below the value at which the ionization started before the current drops back, again discontinuously, to its original low value. If we adopt the view toward which all the evidence of the writer's experience points, that there is no ionization of the mercury vapor of any appreciable magnitude until the electrons have an effective velocity corresponding to 10.3 volts the explanation of the above phenomena is not difficult. Owing to the inelasticity of impact at 4.9 volts, velocities much larger than this value will be relatively infrequent if the vapor pressure is high. As the applied potential increases, however, the probability of an electron's acquiring a velocity of 10.3 volts is increased and more and more of them will actually attain that velocity. As soon, however, as positive ions are formed and are in a position to fall in to the hot wire before recombination, they will, by their impact with the hot wire, cause the emission there of electrons possessing initial velocities considerably higher than the initial velocities of the purely thermal electrons. These high speed electrons will be all the more able to ionize and hence, once started, the ionization will rush to completion. The same reasoning allows us to explain the setting in of ionization, with very hot wires, at applied potentials below the ionizing value. It is only necessary to assume that under these conditions there are a sufficient number of high speed electrons, due to local temperature variations, present to start the ionization. Once started it will be self sustaining.

On the basis of the above theory it is to be expected that as the vapor pressure is diminished it will become more and more possible for an electron to acquire velocities in excess of 4.9 volts, and that finally a point might be reached at which the increase in current due to ionization would take place at values of the applied potential corresponding to the ionizing potential and that the increase in current would be continuous, or nearly so. This was actually found to be the case and was made the basis of an accurate determination of he ionizing potential.

We might also expect, upon the above view, that at any given applied potential, *after* ionization has set in, there would be two definite groups of electrons emitted from the hot wire, one group consisting of electrons having initial velocities corresponding to the thermal conditions of the wire, and the other of electrons whose initial velocities are determined by the potential through which the positive ions have fallen before striking the wire. The initial velocities of this latter group would very probably correspond to potentials differing from the applied potential, at least in cylindrical fields, by a constant amount—the difference depending upon the amount of work which must be done in removing an electron from the surface of the cathode. The existence of these distinct groups of electrons is demonstrated by the

presence of secondary maxima occurring at about 2 volts beyond each principal maximum, beginning with the one at 9.8 volts, on the curves showing the currents to the outside cylinder. No secondary maximum has ever been observed after the first principal maximum at 4.9 volts. Further evidence that these secondary maxima are due to electrons emitted by ionic bombardment of the cathode is furnished by the fact that when conditions are such that the curves for the total current are continuous these maxima disappear or are relatively much less pronounced.

To determine the value of the critical potential at which ionization takes place a series of current potential curves (see curve 1 of the figure) were taken under conditions such that the increase in current was continuous. Under these conditions the increase took place at a very definite potential (in the curve shown, at 12.0 volts). This value must be corrected for initial velocities, of course, and the accuracy of the present determination depends upon the rather high precision with which initial velocity of the electrons could be determined. Simultaneously with the above measurements of the total current, measurements of the current to the outside cylinder against a retarding potential of 3 volts were made. The curves showing the relation of this current to the applied potential show the usual maxima which are very certainly known to occur at *effective* potentials of 4.9 volts, 9.8 volts, etc. (see curve 2 of the figure). If these two current curves are plotted on the same sheet we need only observe the *difference* in potential between the second maximum of the one, at 9.8 *effective* volts, and the sharp break in the other. In the particular example shown this difference is just 0.3 volt giving as the value for the ionizing potential 10.1 volts. The average of five determinations is 10.3 volts and it is believed that this value is accurate to 0.2 volt.

Physical Laboratory,
The University of Minnesota,
April 4, 1917.

The Kinetic Theory of Entropy.[1]

By W. P. Roop.

BOLTZMANN has demonstrated a parallelism between the entropy of an ideal gas and the probability of its molecular state. What is it whose probability increases when we increase the entropy of a gas by imparting heat to it? There is no answer. For in the identification of entropy and probability, an additive constant is neglected. This is found not to remain constant when internal energy or volume suffer change.

Planck, by defining probability otherwise than in the usual mathematical sense, eliminates the additive constant. For proof of the existence of a relation between entropy and probability, however, he relies entirely on Boltzmann. The relation established by Boltzmann is subject to the limitation already mentioned.

[1] Abstract of paper presented at the Stanford meeting of the American Physical Society.

Entropy has a double significance: as a criterion of equilibrium, and as a function of state. The entropy-probability relation appears to hold only in connection with the first of these. Only through the establishment of a general equation of state may kinetic theory be used in calculating entropy as a function of state.

UNIVERSITY OF CALIFORNIA.

ON THE IONIZATION POTENTIALS OF VAPORS AND GASES.

BY J. C. MCLENNAN.[1]

FROM some experiments recently made in Columbia University by Professor Bergen Davis and Mr. Goucher, a preliminary account of which they have kindly sent the writer, it has been amply demonstrated that where electrons having a velocity corresponding to a fall of potential of between 10 and 11 volts are allowed to bombard the vapor of mercury in a high vacuum they are just able to produce in the vapor a definite and distinct type of ionization.

This result constitutes a confirmation of the view presented by the writer[2] in a series of communications, namely, that ionization potentials for the atoms of mercury, zinc, cadmium, magnesium and possibly also for those of other elements are given by the relation $Ve = h\gamma$ where γ is the frequency (1.5, *S*). This view, it should be stated, was reached through experiments made by my students and myself on (I) single line emission spectra, (II) the absorption spectra of metallic vapors, (III) arcing potentials in metallic vapors and (IV) from considerations arising from the theory of atomic structure developed by Bohr.

If we accept the experiments of Bergen Davis and Mr. Goucher as conclusive and as of general application, it follows that ionizing potentials may be calculated for those elements for which the spectral frequency $\gamma = (1.5, S)$ is known. These have been calculated on the basis of $h = 6.585 \times 10^{-27}$ erg. sec., and are given in Table II. In Table I. are collected the ionization potentials for a number of elements which have been found by direct experiment.

TABLE I.

Element.	Ionization Potential.
Helium[3]	20.5 volts.
Neon	16.0 "
Argon	12.0 "
Hydrogen	11.0 "
Oxygen	9.0 "
Nitrogen	7.9 "
Mercury vapor[4]	10.3 "

[1] Abstract of paper presented at the Washington meeting of the Physical Society, April 21, 1917.

[2] McLennan and Henderson, Proc. Roy. Soc., Vol. 91, p. 485, 1915. McLennan, Proc. Roy. Soc., Vol. 91, p. 305, 1915. McLennan, Jour. Franklin Inst., p. 191, Feb. 1916. McLennan, Proc. Roy. Soc., Vol. 92, p. 574, 1916.

[3] Franck and Hertz, Ber. d. Deut. Phys. Ges., Heft. 2, p. 44, 1913.

[4] Goucher, PHYS. REV., N.S., Vol. VIII., No. 5, Nov., 1916. Bergen Davis and Goucher, loc. cit.

TABLE II.

Element.	$\lambda(\gamma = 1.5, S)$.	Ionization Potential Cal. from $\gamma = (1.5, S)$.
Mercury	1187.98 A. U.	10.45 volts
Zinc	1319.95 " "	9.40 "
Cadmium	1378.69 " "	9.004 "
Magnesium	1621.7 " "	7.65 "
Calcium	2028.2 " "	6.121 "
Strontium	2177.5 " "	5.701 "
Barium	2408.0 " "	5.155 "

The values of those wave-lengths of the spectra for the different metals whose frequencies are given by $\gamma = (1.5, S)$ excepting that for barium were taken either from the inaugural dissertation by Dunz, Tubingen, 1911, or from that by Lorenser, Tubingen, 1913. The value $\lambda = 2{,}408$ for barium was deduced by the writer from the frequency $\gamma = (1.5, S)-(2, p_2)$ which has been estimated to be that of $\lambda = 8210.63$. The experiments of Bergen Davis and Mr. Goucher which have been referred to above go to show that the ionization in mercury vapor which was observed by Franck and Hertz with electrons possessing kinetic energy corresponding to 4.9 volts was not due to a direct ionization of the atoms of the vapor, but had its origin in the metallic electrodes of the ionization vessel and was due to the photo-electric action of the light of wave-length $\lambda = 2{,}536.72$ which was emitted by the mercury atoms under the bombardment of the electrons.

Although the experiments have been arranged with much skill and appear to be conclusive, another factor has recently been introduced by some experiments of one of my students, Mr. R. C. Dearle. In these he has found that with the vapors of mercury, zinc, and cadmium there is very marked absorption at the wave-length whose frequency is given by $\gamma = (2.5, S)-(2, P)$. It is not easy to interpret this absorption but it should be remembered that in the spectra of the elements mentioned the wave-length which possesses the greatest amount of energy is the one of frequency $\gamma = (2.5, S)-(2, P)$.

One inference on the Bohr theory which may be drawn from this result is that the frequency $\gamma = (2.5, S)$ with the relation $Ve = h\gamma$ should give *an ionizing potential.* Values calculated on this assumption are given in Table III.

TABLE III.

Element.	$\lambda(\gamma = (2.5, S) - (2, P))$.	$\lambda(\gamma = 2.5, S)$.	$V = \frac{h(2.5, S)}{e}$.
Mercury	10,140.58 A. U.	4,937.4	2.51 volts
Zinc	11,055.4 " "	5,007.0	2.48 "
Cadmium	10,395.17 " "	5,201.7	2.39 "
Magnesium	11,828.8 " "	5,506.3	2.25 "
Calcium	13,038.0 " "	5,562.5	2.23 "
Strontium[1]	13,717.82 " "	5,902.8	2.10 "
Barium[1]	13,251.00 " "	6,283.2	1.98 "

[1] Estimated.

The value for mercury, it will be seen, is approximately 2.5 volts, which is just one half the value obtained by Franck and Hertz.

At the New York meeting of the A. A. A. S. Professor Millikan described some experiments made in his laboratory in which arcs were obtained in mercury vapor with applied potentials of between 3 and 4 volts. Up to that time arcs had not been obtained by the writer or his students in mercury vapor, with less than an applied potential of between 10 and 11 volts, but the point has now been reëxamined by my students and myself, and we have found that with mercury and cadmium vapors it is possible to strike arcs with applied potentials less than those given by the relation $V = h.\ (1.5,\ S)/e$ if tungsten filaments be used which are heated much higher than those used in our first experiments.

Moreover with mercury vapor it has been found possible to sustain an arc with applied potentials as low as 3.3 volts and with cadmium vapor with applied potentials as low as between 2 and 3 volts.

It is difficult to interpret the fact that arcs can be maintained in vapors with such low voltages, but it is possible that it may have something to do with the results set forth in Table III. An initial emission of a radiation stimulates by electronic bombardment combined with a photo-electric effect due to the radiation reacting on the incandescent cathode might account for it. To elucidate the matter, however, additional experimental work will have to be carried out.

In Professor Bergen Davis' communication to me he pointed out that he and Mr. Goucher had found that when mercury vapor was bombarded by electrons, having a velocity corresponding to 6.7 volts it emitted light of wave-length $\lambda = 1849.6$. In this connection it may be of interest to recall that in a paper by Mr. Thomson and myself,[1] recently published, it was pointed out that when cadmium vapor was fed into a gently burning Bunsen flame the vapor was found to emit light of wave-length $\lambda = 3{,}260.17$ but that when the flame was made to burn strongly, light of wave-length $\lambda = 2{,}288.79$ was also emitted.

These two lines it will be noted are series lines analogous to the lines $\lambda = 2{,}536.72$ and $\lambda = 1{,}849.6$ in the mercury spectrum.

Mr. Ireton, one of my students, and I have also found recently that with cadmium vapor heated in an exhausted quartz tube and bombarded by electrons, whose speed is gradually increased, the line $\lambda = 3{,}260$ comes out with speeds of about 4 volts, but that when potential differences of about 6 volts are applied the line $\lambda = 2{,}288.79$ comes out as well on the photographic plates. In this regard it will be seen that cadmium and zinc vapors act under electronic bombardment in a manner analogous to mercury vapor, as Messrs. Bergen Davis and Goucher have shown.

It was also found that with zinc vapor the line $\lambda = 3{,}076$ came out when the electrons attained a speed of about 4 volts and that when the electronic

[1] McLennan and Thomson, Proc. Roy. Soc., Vol. 92, p. 584, 1916.

speeds reached a value somewhere near 6 volts the line $\lambda = 2{,}139$ also came out strongly.

THE PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
April 7, 1916.

OPTICAL CONSTANTS BY REFLECTION MEASUREMENTS.[1]

BY L. B. TUCKERMAN, JR., AND A. Q. TOOL.

THE properties of the light reflected from metals has been used as the best available method of determining their optical constants. These measurements, however, are vitiated by the effect of surface films, or transition layers. L. Lorenz and Drude and, in a more complete form, Maclaurin, have developed the theory of isotropic surface films, and Hebeker and Schulz of doubly refracting surface films. Maclaurin has corrected Conroy's determinations of the optical constants of steel from its reflection coefficients using his theory to eliminate the effect of the surface film, but his results are subject to an error of several per cent., and of course allow of no independent check.

In trying to develop a practical method for eliminating the effect of surface films from the measurement of optical constants by reflection it seemed best to work first with transparent media where spectrometer measurements afforded a check on the adequacy of the reflection method.

Using an old flint glass prism ($\nu = 1.6537$) whose surface showed a slight deterioration and produced marked ellipticity, we were able to determine the refractive index within 1/5 per cent. from reflection measurements, although the value calculated by Brewster's formula using the principal angle of incidence as the polarizing angle was 4½ per cent. too small.

The measurements indicate that in the region from 40° to 70° incidence the effect of the surface film is adequately represented by a modified Drude theory—similar to Maclaurin's theory, involving absorption in the surface film, but neglecting second order effects. Below 40° and above 70° there seems to be a systematic deviation which could be accounted for by second order effects, if the accuracy of the determinations of the constants in that region warranted it.

Drude's original formula

$$\frac{R_p}{R_s} = -\frac{\cos(\vartheta+\chi)}{\cos(\vartheta-\chi)}\left[1 + i.4\bar{\rho}\,\frac{\nu^2}{\sqrt{1+\nu^2}}\,.\,\frac{\sin\vartheta\,tg\,\vartheta}{tg^2\vartheta-\nu^2}\right]$$

was replaced by

$$\frac{R_p}{R_s} = -\frac{\cos(\vartheta+\chi)}{\cos(\vartheta-\chi)}\left[1 + i(a-ib)\,\frac{\sin\vartheta\,tg\,\vartheta}{tg^2\vartheta-\nu^2}\right],$$

where (a) and (b) may be called the refractive and absorptive constants of this film. For transparent films b should equal zero.

UNIVERSITY OF NEBRASKA.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

A NEW THEORY CONCERNING THE MATHEMATICAL STRUCTURE OF BAND SERIES.[1]

BY RAYMOND J. BIRGE.

ACCORDING to Deslandres' law for band series, if the first frequency difference of successive lines ($\Delta\nu$) be plotted against integral values of "m" there should be obtained a straight line either through the origin or very close to it. The only serious modification of this theory, thus far proposed, namely that by Thiele, has in the author's opinion been disproved definitely by Uhler. In the case of the long (A_1) series running from the first head of the 3883 CN band (the longest band series known) the author has found that the curve in $\Delta\nu$ and m is accurately a hyperbola running through the origin, with its real axis (on the particular scale of units used) making an angle of 13° + with the $\Delta\nu$ axis.

The final relation between frequency and m is most easily obtained by actually taking the successive sums of the $\Delta\nu$'s. The agreement thus obtained is very satisfactory. For the 164 known lines of the A_1 series the average difference obs.-calc., with the "perturbations" included, is only .005 Å. Without these it is only one half as great. This is a greater accuracy than has ever been obtained for any band series formula.

The hyperbola through the origin has four undetermined coefficients. The final equation in frequency thus has five constants. The equation recommended by Kilchling, in ascending even powers of m, if carried out to five constants, is not nearly so satisfactory, but is unquestionably the best interpolation formula that can be used.

The corresponding series from the second and third heads give hyperbolæ of almost identical size and shape, but differently located. The integral of the hyperbola, while not at all the proper functional form such that its *finite* first differences (*not* its derivative) shall give a hyperbola, agrees to within about .005 Å. with the actual $\Sigma\Delta\nu$ at all points in the series.

According to the above theory, a band series, if extended, will end when the hyperbola again cuts the m axis, and would thus have a "tail" having the same general appearance as the "head," *i. e.*, a finite number of lines in a finite space, whereas Thiele's "tail" required an infinity of lines. Thus weak band heads may be the "tails" of strong bands running in the opposite direction. The author personally does not believe in the existence of such tails, as all series continually decrease in intensity, after a certain point, those studied ending in each case at about $m = 168$. If continued to the tails they would have 220, 208, and 192 members, respectively, and would end at about 3567.2 Å., 3608.8 Å., and 3647.5 Å. There are no known tails at these points.

It is not claimed, at this time, that the above formula will hold for all band

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

series. It seems probable, however, that it will be found to hold for all series of the type of which those mentioned above are the best known and most accurately measured examples.

DEPARTMENT OF PHYSICS,
SYRACUSE UNIVERSITY.

GENERALIZED COÖRDINATES, RELATIVITY AND GRAVITATION.[1]

BY E. B. WILSON.

PROFESSOR WILSON discusses the relation between generalized or curvilinear coördinates, the relativity of Newtonian mechanics, the relativity of electrodynamics, and the new gravitation theories of Einstein. The paper will appear in the Astrophysical Journal, 1917.

THE MOTION OF AN AËROPLANE IN GUSTS.[1]

BY E. B. WILSON.

CONTINUING earlier work (Proceedings National Academy of Sciences, Washington, 2, 1916, 294–297) Professor Wilson treats periodic longitudinal gusts, gives a general discussion of resonance as applied to aëroplane problems, takes up the limiting conditions arising in infinitely sharp gusts; then turning to the lateral motion, determines the effect of single or periodic side gusts and of yawing and rolling gusts. As the rolling gust is sometimes serious because of its tendency to put the machine into a spiral dive, the motion of a machine constrained to fly without rolling is treated and is shown to be dynamically stable. The paper will be presented in detail to the American Philosophical Society and offered for publication in their Proceedings.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.

THE SIGNIFICANCE OF CERTAIN NEW PHENOMENA RECENTLY OBSERVED IN PRELIMINARY EXPERIMENTS ON THE TEMPERATURE COEFFICIENT OF CONTACT POTENTIAL.[1]

BY A. E. HENNINGS.

FROM the results obtained in these preliminary experiments it must be concluded that in the case of none of the experimental work on the temperature coefficient of contact potential hitherto recorded can it be positively asserted that what has been measured is the *true* coefficient. The measured contact potential difference in vacuo between two copper plates, the temperature of one of which was varied through a wide range, was found to undergo abrupt as well as gradual changes. The nature and extent of these changes depend on the degree of exhaustion of the containing vessel in such a way as to indicate that gas films either mask the true effect or are themselves the active agents which account for the phenomenon of contact potential.

UNIVERSITY OF SASKATCHEWAN.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

NATURAL AND MAGNETIC ROTATION AT HIGH TEMPERATURES.[1]

BY FREDERICK BATES AND F. P. PHELPS.

PRECISION measurements have been secured for both the natural and magnetic rotation of the plane of polarization in quartz for a temperature range from 20° C. to 1000° C. Quartz recrystallizes at about 575° C. changing from α to β quartz. The curve for the natural rotation makes a right angle turn at this point and shows a discontinuity. In contrast to the natural rotation the magnetic rotation shows no change at 575° and has no temperature coefficient. Similar measurements for the magnetic rotation of iron films have also been made.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A MEASURING ENGINE FOR READING WAVE-LENGTHS FROM PRISMATIC SPECTROGRAMS.[1]

BY L. G. HOXTON.

THE reduction of prismatic spectrograms is usually carried out by mean of Hartmann's dispersion formula,

$$\lambda = \lambda_0 + \frac{c}{s_0 - s},$$

especially for the routine work of astronomical observatories. Here λ is the wave-length, s the micrometer setting, while λ_0, s_0 and c are empirically determined constants, three wave-lengths at least being known.

The use of the formula, which involves computations with seven significant figures, is laborious, even where a calculating machine is available, for the evaluation of the last term involves the operation of division.

The present paper proposes a mechanical solution of this formula simultaneous with the procedure of setting on the lines of the spectrum plate. The numerical results herein given must be regarded as preliminary because the apparatus was crude for the degree of precision involved and hurriedly assembled; but the accuracy actually attained is sufficient, in the author's opinion, to show that no great difficulty will be encountered in constructing an inexpensive attachment to any good measuring engine that would enable one to read off wave-lengths with a precision considerably exceeding the precision of setting upon the best of lines on a photographic plate.

The principle involved is that of geometrical projection. Use of this principle for the graphical solution of the Hartmann formula has already been proposed[2] and its mathematical exposition is, therefore, omitted here.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] F. Henroteau, "On a Graphical Construction for Obtaining the Wave-lengths in Prismatic Spectra," Monthly Notices Roy. Ast. Soc., LXXVII., 1, p. 77.

The engine is an assembly of two interferometer beds (the mirrors being removed) and a framework capable of rotating about an accurate pivot and connected to one of the interferometer screws in such a way that settings made upon the spectrum by means of this screw will move a hair-line fixed to the framework into a position such that, when a setting is made upon this hair-line by means of the other screw the latter screw will read off wave-lengths. The adjustment of the engine for any given region of the spectrum requires about a half-hour, and when once made, need not be repeated.

A trial was made in which the measures of 53 lines, previously made with another engine and reduced by the Hartmann formula, were reproduced on the setting screw of this engine, while the wave-lengths were read off as above described. The results obtained by the two methods were then compared. The differences, expressed in Ångstrom units, are as follows classified according to magnitude regardless of sign.

Difference-Range. Å. U.	Number of Instances.
From .000 to .010 inclusive	39
" .011 " .020 "	7
" .021 " .030 "	5
" .031 " .040 "	1
" .041 " .050 "	1
	53

The plate concerned was taken with a 3-prism spectrograph of the Yerkes Observatory and covered a range from λ 4,434.168 to λ 4,617.452.

Especial attention is called to the fact that this test involves the errors of both screws, while for the direct measurement of a plate, the errors of but one screw are involved. Further, the screws here employed were of a quality inferior to that demanded by such exacting work. It is proposed to continue this work with a fine screw in the near future.

UNIVERSITY OF VIRGINIA,
March 8, 1917.

THE WAVE-LENGTH OF LIGHT FROM THE SPARK WHICH EXCITES FLUORESCENCE IN NITROGEN.[1]

BY CHARLES F. MEYER.

IT was shown by Professor Wood in 1910 that radiations from the spark were capable of exciting fluorescence of air and other gases. Subsequent investigations by Wood & Hemsalech, and Meyer & Wood, indicated that the fluorescence was excited by light beyond the Schumann region. The wave-length could not be determined, however, and some doubt still existed as to whether the exciting light was really shorter than the limit reached by Schumann. A method has been found of determining the wave-length of the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

exciting light. A small grating is placed immediately below a fine slot in one of the spark terminals, and the fluorescence in the path of the direct and diffracted beams is photographed. From the angle between the two beams the wave-length is calculated. Three determinations have been made with two gratings which give a wave-length of in the neighborhood of 1,450 Å. for the light which excites the fluorescence of the water band 3,064 Å. in nitrogen containing a trace of water vapor. An interesting question arises concerning the transparency of the air near a spark for this region of the spectrum.

UNIVERSITY OF MICHIGAN.

THE NECESSARY PHYSICAL ASSUMPTIONS UNDERLYING A PROOF OF PLANCK'S RADIATION LAW.[1]

BY RUSSELL V. BICHOWSKY.

IN order to prove Planck's radiation law by means of the quantum theory only two physical assumptions need be made, first, that energy is absorbed or radiated by a radiating system in quanta of $h\nu$, second that a radiating system has the statistical properties of a perfect gas, *i. e.*, that Maxwell's distribution law holds for the distribution of the local values of the energy among the coördinates defining the state of the radiating system. (The usual auxiliary assumptions such as Planck's oscillators, or Larmor's regions of equal probability are not only unnecessary but misleading.)

Although these two assumptions are sufficient for deriving the Planck radiation law both of them, and particularly the last, are very dubious, it being almost unthinkable that a radiating system can have the statistical properties of a perfect gas and yet not have the equipartition law hold. For these and other reasons it seems necessary to give up at least the second of the quantum hypotheses and to assume that the distribution of energy in a radiating system does not obey Maxwell's law, that is, that in a radiating system the distribution of the local values of the coördinates is a function not only of the total energy of the system but also of some other variable. If we do this and assume for definiteness that the distribution of the local values of the generalized momenta is a function not only of the total energy E of the system but also of the Helmholtz free energy A, and further assume that the total energy of a radiating system cannot be less than a certain limiting value E_0 (E_0 turns out to equal $h\nu$), we can, following the methods of Gibbs and Ratnowsky, derive in a very simple manner the Planck radiation law and moreover we can do this without assuming discreteness of radiant energy, without contradicting classical mechanics (equipartition does not hold for systems of this kind), without discarding infinitesimal analysis or without contradicting thermodynamics or the direct experimental evidence of the photoelectric effect that the $h\nu$ law holds only as a limiting case.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

The Measurement of "h" by Means of X-Rays.[1]

By F. C. Blake and William Duane.

At the New York meeting (February, 1917) of the American Physical Society, we presented a preliminary paper on the measurement of "h" by means of X-rays. In this paper we stated that the values of "h" obtained depended to some extent upon the interpretation placed upon the shapes of the experimental curves. We have now succeeded in determining accurately the corrections that must be applied to the measurements and have obtained results that are quite consistent with one another.

The chief corrections appeared to be the following: (*a*) A correction for the widths of the slits and of the source of the rays. The value of this correction we obtained by measuring the ionization currents in the instrument for several positions of the micrometer screws that closed the slits.

(*b*) A correction for the depth of penetration of the X-rays into the reflecting crystal. This correction we obtained by making the ionization chamber slit very narrow, and, with the crystal in a fixed position, measuring the breadth of the X-ray beam by moving the ionization chamber. If this is done on both sides of the zero line of the instrument, we get values for both the ionization chamber angle and the crystal angle. The former should be twice the latter, if what we may call the effective reflecting plane of the crystal coincides with the axis of rotation. As a matter of fact, this never coincides with this axis for all wave-lengths, the effective reflecting plane lying at a different distance below the surface of the crystal for different wave-lengths. By taking the difference between the measured values of the ionization chamber angle and of twice the crystal table angle, we calculate the distance of the effective reflecting plane from the axis of rotation, and from this we get a correction that must be applied to the measurements of the wave-lengths.

In estimating the ionization chamber angle, we measure from the center of the X-ray beam on one side to the center of the X-ray beam on the other. If we take the distance between the two maxima, we get inconsistent results, for the maxima do not, in general, coincide with the center of the beam on account of the difference in the distribution of energy in the spectrum for different wave-lengths.

We now have eleven measurements of the values of "h," determined by using six slightly different methods of taking the data. These methods may be briefly stated as follows:

1 and 2. In these methods the ionization chamber slit is very wide and there are two slits between the crystal and the X-ray tube, each of which is narrow, thus permitting a very narrow beam of X-rays to strike the crystal. In method 1, the part of the curve representing the X-ray intensity as a function of the voltage, which is nearly straight, is extrapolated, and the point at which the extrapolated curve cuts the axis of zero intensity is taken as the

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

point at which the X-rays of wave-length corresponding to the center of the beam vanish. No slit correction is needed in this case. In method 2, however, we take the point at which the curve actually vanishes, and make a correction for the widths of the slit and source. These two methods give the same result to within less than one tenth of one per cent. In methods 3 and 4, we use only one narrow slit between the X-ray tube and the crystal, and make the ionization chamber slit very narrow also. The interpretations of the curves in methods 3 and 4 are similar to those in methods 1 and 2 respectively. As a matter of fact, slit corrections in these methods do not turn out to be quite as large as in methods 1 and 2, but there is a certain disadvantage, in that the position of the effective reflecting plane in the crystal must be determined with considerable accuracy.

Methods 5 and 6 do not differ from methods 3 and 4 except that we use the curves representing the ionization currents as functions of the wave-length at constant voltage, instead of the curves representing the ionization currents as a function of the voltage at constant wave-length.

The following table contains the values of "h" obtained by the above methods:

Methods.	Values of "h."	Methods.	Values of "h."
1	6.555×10^{-27}	5	6.557×10^{-27}
2	6.555×10^{-27}	5	6.548×10^{-27}
3	6.555×10^{-27}	6	6.555×10^{-27}
3	6.557×10^{-27}	6	6.554×10^{-27}
4	6.560×10^{-27}	6	6.554×10^{-27}
5	6.557×10^{-27}	Mean	6.555×10^{-27}

The weighted mean value of "h" contained in the above table does not differ very much from those determined in other ways; for instance, from the distribution of energy in the black body spectrum, using Planck's radiation formula; or from the photo-electric effect, using the Einstein equation and Millikan's experimental results; or from Bohr's formula for the Rydberg constant, using the data obtained from spectrum analysis. The values of "h" in the table above have been calculated on the basis of $e = 4.774 \times 10^{-10}$ (Millikan's value). Using the same value of e and $e/m = 1.770 \times 10^{7}$ we calculated from Bohr's equation $h = 6.544 \times 10^{-27}$. According to Millikan's photo-electric experiments, the value of $h = 6.57 \times 10^{-27}$, and this seems to fit the black body radiation data very well. Earlier experiments with X-rays have given for "h" the values 6.51×10^{-27} (Duane and Hunt), 6.53×10^{-27} (Webster), and 6.59×10^{-27} (Hull).

In making the above determinations of the value of "h," we measured the voltage applied to the tube, by means of an electrostatic voltmeter that we calibrated by means of a current flowing through a high resistance, as in the previous experiments. The current supplied to the X-ray tube came from a high potential storage battery.

Any error in the estimation of the crystal spacing produces, of course, a corresponding error in our value of "h." We have taken for the grating space (calcite crystal) the value 3.027×10^{-8} cm.

HARVARD UNIVERSITY.

THE REFLECTION COEFFICIENT OF MONOCHROMATIC X-RAYS FROM ROCK SALT AND CALCITE.[1]

BY A. H. COMPTON.

ACCORDING to classical electrodynamics, the ratio of the energy of a beam of X-rays reflected from a crystal to that of a beam incident upon it is given by Darwin's formula (2):

$$(1) \qquad R = \frac{E_r}{E_i} = \frac{1}{\Delta\theta} \cdot \frac{N^2\lambda^3}{2\mu} \cdot \frac{1 + \cos^2 2\theta}{4 \sin\theta \cos\theta} \left(\frac{e^2}{mC^2}\right) e^{-B \sin^2\theta}$$

In this expression E_r is the energy in the beam of X-rays of wave-length λ which is reflected at a glancing angle θ, while the crystal is rotated with uniform angular velocity through an angle $\Delta\theta$ which is large enough to include all angles at which any appreciable amount of rays of this wave-length are reflected. E_i is the total energy of wave-length λ which falls on the crystal during this time; N is the number of electrons per unit volume in the crystal. μ is the absorption coefficient of the X-rays in the crystal; e is the charge and m the mass of an electron and C is the velocity of light. The factor ψ depends upon the distribution of the electrons in the atoms of the reflecting crystal, its value being approximately 0.76 for the first order reflection from the cleavage planes of rock salt, and 0.75 for calcite.[3] The constant B depends upon the thermal motion of the atoms of the crystal. It may be taken to be 2.6 in the case of rock salt reflecting molybdenum α rays, and 0.18 in the case of calcite.

It is evident that by measuring the ratio of the reflected energy E_r of wave-length λ to the incident energy E_i of the same wave-length, a test of this formula may be made. In order to measure this ratio, monochromatic X-rays were obtained by the reflection of a beam of X-rays from a crystal mounted on a standard Bragg X-ray spectrometer. The source of X-rays was a Coolidge tube with a molybdenum target, kindly supplied by Dr. Coolidge, so that it was possible to obtain a comparatively intense beam of monochromatic X-rays of wave-length 0.721×10^{-8} cm. The monochromatic beam thus obtained was reflected in turn by a crystal mounted on a second spectrometer, and the intensity of the second reflection was determined by the ionization method. This was compared with the intensity of the beam

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] C. G. Darwin, Phil. Mag., 27, 325 (1914) and 27, 675 (1914).

[3] A. H. Compton, PHYS. REV., 9, 29 (1917).

incident on the second crystal by removing the crystal and swinging around the ionization chamber, so as to receive directly the monochromatic beam.

The quantity E_r was measured by the total deflection of the electrometer when the second crystal was turned with constant angular velocity through an angle $\Delta\theta$ past the angle of maximum reflection. The corresponding value of E_i was the deflection produced by the monochromatic beam when it passed into the ionization chamber for a time equal to that required to move the crystal through the angle $\Delta\theta$. The average value of R obtained in this manner was 0.0050 ± 0.0003 deg.$^{-1}$ in the case of the reflection from a cleavage face of calcite, and 0.023 ± 0.001 deg.$^{-1}$ for a cleavage face of rock salt.

If in equation (1) N is calculated assuming each atom to possess a number of electrons equal to its atomic number, and μ is taken to be the usual absorption coefficient (calcite 23.5: rock salt 18) we obtain R for rock salt = 0.040 deg.$^{-1}$ and for calcite 0.058 deg.$^{-1}$. It will be seen that for rock salt the experimental value of this ratio is about one half the calculated value, and for calcite is less than one tenth as large. The reason for this discrepancy is doubtless due to the fact that at the angle of maximum reflection a selective absorption occurs, as has been predicted by Darwin[1] from theoretical considerations, and has been observed experimentally by W. H. Bragg[2] in the case of diamond. The plausibility of this explanation is increased by the fact that if the reflecting surface of a calcite crystal is roughened by grinding, the reflection coefficient is some three times as great as from a cleavage face. The grinding makes the surface of the crystal imperfect, and thus greatly reduces the selective absorption.[1] Experiments are in progress to make quantitative measurements of the effective absorption coefficient at the angle of maximum reflection.

UNIVERSITY OF MINNESOTA.

ON THE OCCURRENCE OF HARMONICS IN THE INFRA-RED ABSORPTION SPECTRA OF GASES.[3]

BY W. W. COBLENTZ.

UNDER this title, Kemble[4] has given a theory to account for the occurrence of absorption bands in harmonic series, in certain gases.

The observations, made by the writer, on H_2S, SO_2 and NH_3, being exceptions to his rule, Kemble concludes that the "extra lines may be due to impurities or they may be due to the nonlinearity of the law of force."

Following the procedure well known to spectroscopists (viz., if impurities are suspected, to identify their absorption bands) it is found that these numerous absorption bands cannot be accounted for on the basis of impurities. This is very conspicuous in SO_2 which has very strong absorption bands. In the

[1] C. G. Darwin, Phil. Mag., 27, 675 (1914).

[2] W. H. Bragg, Phil. Mag., 27, 881 (1914).

[3] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[4] PHYS. REV., 8, p. 701, 1916.

case of NH_3, Baly[1] on the basis of a different theory computes *sixteen* absorption bands, between 3 and 14 μ, *eleven* of which coincide closely with observed bands.

Water is one of the most unusual substances known, as regards its absorption bands. The absorption spectrum of water vapor consists of many fine lines which, for water in the liquid state, coalesce into bands, the most conspicuous of which occur at 1, 1.5, 2, 3, 4.75 and 6 μ. The apparent harmonic relation of the centers of gravity of the wide bands, comprising groups of these absorption lines is probably accidental. At least, one would hardly consider them in connection with the closely harmonic absorption bands of the simpler spectra of other substances.

From a consideration of various phases of the problem of harmonics among absorption bands the writer has come to the conclusion that the lack of agreement is as much (if not more) the fault of the theory as it is of impurities in the material examined.

BUREAU OF STANDARDS,
WASHINGTON, D. C.
April 20, 1917.

THE USE OF A THOMSON GALVANOMETER WITH A PHOTOELECTRIC CELL.[2]

BY W. W. COBLENTZ.

THE development of a photoelectric cell by Kunz[3] giving a direct proportionality of response with variation in intensity of the light stimulus provides a simple instrument for investigations in the blue, violet and ultra-violet parts of the spectrum, where the thermopile is operated with difficulty.

The object of the present communication is to call attention to the usefulness of a high resistance iron-clad Thomson galvanometer, instead of an electrometer, in connection with the above-mentioned photoelectric cell.

Tests were made upon a two-coil instrument having a resistance of 1,300 ohms. Using a single swing of only 2 seconds and scale at 2 m. the current sensitivity was $i = 2.7 \times 10^{-11}$ amp. A four-coil instrument, of 5,300 ohms. and a heavy suspension, under similar conditions had a sensitivity of $2 = 6.2 \times 10^{-11}$ amp., or 8×10^{-13} amp. for a resistance of 1 ohm. From this it is evident that a sensitivity of 1×10^{-12} amp. is easily attained, which is far greater than would be required for transmission spectra investigations in the blue violet and ultra-violet.

BUREAU OF STANDARDS,
WASHINGTON,
April 20, 1917.

[1] Astrophys. Jr., 42, p. 66, 1915.

[2] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[3] Astrophys. Jr., Mar., 1917.

THE HIGH FREQUENCY ABSORPTION BANDS OF SOME OF THE ELEMENTS.[1]

BY F. C. BLAKE AND WILLIAM DUANE.

IT has long been known that a marked change in the absorption of X-radiation takes place at a frequency near that of the characteristic rays. On the high frequency side of the characteristic rays of an element, the element absorbs to a much greater extent than on the low frequency side. Some time ago one of us[2] presented a paper to this society which contained evidence showing that the sharp break in the absorption curve occurred at a frequency near that of the beta line of the K X-ray series, rather than in the neighborhood of the alpha line, notwithstanding the fact that the alpha line contained much more radiated energy than the beta and is of lower frequency. Subsequently, Professor Bragg[3] obtained more accurate data, and came to the conclusion that the break occurred above the beta line, and in the neighborhood of the frequency of the gamma line.

The experiments that the authors of this paper wish to report to the society had for their object to determine the relation between the frequency at which the change of absorption takes place and the atomic number or atomic weight of the element. Investigations on this point have been carried on by deBroglie, and by Wagner by methods that differ somewhat from ours.

In another paper presented at this meeting, we have shown that measurements of X-ray wave-lengths in which the angle measured to the ionization chamber slit or to a line on a photographic plate is used in making the calculations, are not reliable, unless great care is taken to correct for the penetration of the rays into the reflecting crystal, and especially for the change of this penetration with changing wave length. In the experiments described in that paper, we used two distinct methods of taking the readings. In one method the X-ray spectrometer slit in front of the ionization chamber and that between the crystal and the X-ray were were both very narrow. In this method we had to correct carefully for the change of penetration of the X-rays into the crystal with changing wave-length. In the other method two very narrow slits, both lying between the crystal and the X-ray tube defined a narrow beam of X-rays, the whole portion of which that was reflected from the crystal passed into the ionization chamber, the ionization chamber slit being sufficiently wide for the purpose. In this method, the angle between the position of the reflecting planes on one side and the position of these planes when the reflection takes place on the other side is used to calculate the wave-length, and no correction need be made for the penetration of the rays into the crystal.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 20–21, 1917.

[2] The Relation between the Wave-length and Absorption of X-rays, by William Duane, October, 1914.

[3] Phil. Mag., March, 1915.

We used the latter method in determining the wave-length at which the change of absorption takes place. The element, or salt of the element in a powdered form was fastened by adhesive plaster in front of the slit nearest the X-ray tube. A high potential storage battery furnished the current for the X-ray tube and we kept the voltage constant, by gradually changing a water resistance, observing the voltage as indicated by an electrostatic voltmeter. This instrument was so sensitive that a change in the voltage of one part in seven thousand could easily be detected. We then measured the current in the ionization chamber for different angles of the crystal proceeding in the neighborhood of the absorption band by steps of one minute. The change in absorption was very marked in most of the experiments. It occurred within a range of about two minutes and fifty seconds of arc. We think that the center of this range represents the critical wave-length, and that we can estimate this center to within about one quarter of a minute. Since the smallest double angle measured was six degrees, this means that the relative accuracy of the measurements is less than a tenth of one per cent.

The measurements that we report today refer to the K series of characteristic X-rays of the elements. The data obtained by Moseley in his classic investigation of the alpha and beta lines indicate that the square root of the frequency does not increase by exactly equal amounts from element to element. The change in the square root of the frequency for the elements that he investigated above zinc (atomic number 30) is somewhat larger than for those below. Other investigators have also found this to be true. The curve representing the square root of the frequency as a function of the atomic number has a slight curvature upward for elements above bromine. DeBroglie's experiments on the absorption of X-rays show the same characteristic.

The thought occurred to us that perhaps a part at least, if not all of this departure from the straight line law might be due to the fact that the data obtained had not been accurately corrected for the change of penetration of the X-rays into the crystal with changing wave-length, and it was for this reason, largely, that we undertook the experiments we now wish to report on. The following table contains a summary of our measurements:

Element.	Atomic Number.	Absorption Wave-length.	Square Root of Frequency.	Wave-length deBroglie.	Differences Per Cent.
Cerium	58	$.3067 \times 10^{-8}$	3.127×10^{9}	$.2979 \times 10^{-8}$	− 2.87
Barium	56	$.3306 \times 10^{-8}$	3.012×10^{9}	$.3249 \times 10^{-8}$	− 1.72
Iodine	53	$.3725 \times 10^{-8}$	2.838×10^{9}	$.3666 \times 10^{-8}$	− 1.58
Antimony	51	$.4063 \times 10^{-8}$	2.717×10^{9}	$.3985 \times 10^{-8}$	− 1.92
Tin	50	$.4246 \times 10^{-8}$	2.660×10^{9}	$.4188 \times 10^{-8}$	− 1.37
Cadmium	48	$.4631 \times 10^{-8}$	2.546×10^{9}	$.4580 \times 10^{-8}$	− 1.10
Palladium	46	$.5073 \times 10^{-8}$	2.432×10^{9}	$.5029 \times 10^{-8}$	− 0.87
Molybdenum	42	$.6178 \times 10^{-8}$	2.204×10^{9}	$.6113 \times 10^{-8}$	− 1.05
Zirconium	40	$.6869 \times 10^{-8}$	2.090×10^{9}	$.6813 \times 10^{-8}$	− 0.82
Bromine	35	$.9176 \times 10^{-8}$	1.808×10^{9}	$.9139 \times 10^{-8}$	− 0.40

It will be seen from the data of the last column that there is a progressively increasing difference between our values and those of deBroglie, as we proceed upward from element to element above bromine. This difference equals approximately the error that one makes in measuring wave-lengths, if the axis of rotation passes through the front surface of the crystal instead of through the effective reflecting plane.

The curve representing the square root of the frequency as a function of the atomic number plotted from our data is less inclined to the atomic numbers axis than that plotted from deBroglie's data, but even our curve differs from a straight line by slightly more than what we regard as the error of our measurements.

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